PART II of   Simulating a Variable Speed Centrifugal Compressor

The Simulation Modeling approach has been described in PART I and visualized in Block Flow Diagram – 4 showing two sub-models : an ideal (isentropic) compression process performed inside the compressor machine plus an Energy – or 'Power-Loss' process proceeding concurrently in the real, actual machine. The Simulation Model fits on (part) of a single worksheet of an Excel Spreadsheet. The compression model includes the accounting for the compression of a non-deal gas, i.e. including the gas compressibility factor 'Z'. The energetic losses have in PART I been described by a newly developed correlation for the 'adiabatic efficiency factor', 'Eta-ad' allowing the actual, real shaft power of the compressor machine to be calculated. In this PART II a new sub-model is developed to describe and calculate the (shaft) Power Losses i.e. that part not converted into a pressure rise but into a temperature rise! With this new Power Loss sub-model the (total) shaft power requirement can be calculated without the aid of an efficiency factor!

In PART I it was pointed out that from the 'fingerprint' Head - Flow curve, the H* - Q* curve, the surge points of the original manufacturer's performance curve are reduced into *a single point.* In a simulation the surge points and the surge line can be generated from this reduced point by doing the transformation in reverse, however no details were given. In this PART II a detailed description of calculating the surge points and line in a simulation from the coordinates of the reduced, single point of surge is given. In addition considerations about the choke points and line are being presented. It should be noted that the compressor simulation presented here does not require any input of detailed machine construction details neither external nor internal, except for the presence of any inter-stage cooling. This post is long and is divided into the following parts:

'1 Describing the principles behind the performance curves: a summary.
'2 Fingerprinting the Characteristic Performance (Model Identification).
'3 Describing the Surge Points and Surge Line in the Performance Diagram.
'4 Describing the Choke Points and Choke Line.
'5 Notes on the Shape of the Fingerprint Curve ' H*fp '
'6 The Gas Power 'Gad' and the Unified 'G*ad' Curve.
'7 The Shaft Power 'Ps' and the Adiabatic Efficiency Factor 'Eta-ad' .
'8 Fingerprinting the Overall Power-Loss Process(es)
     '8.1 Determination of the losses during operation of compressor "vscCR-01" .
     '8.2 Pressure loss and Power Loss.
     '8.3 Characterizing the Overall Loss Process.
     '8.4 Fingerprinting the Overall Loss Process(es).
     '8.5 Predicting the Power Loss.
'9 Predicting the Shaft Power in the Simulation of 'vscCR-01"
'10 Upgraded and updated Simulation Excel Spreadsheet. The simulation consists of
     three parts on the same worksheet:
     Part 1: Input data: Compressor Performance data ; Gas Quality data
     Part 2 (A): Analyzing the Manufacturers data: calculation of Q* and H*
     Part 2 (B) -1: 'Fingerprinting' the Performance Curves : H*fp
     Part 2 (B) -2: Determination of the Surge – and Choke Points and Lines
     Part 3: Simulation of Compressor "vscCR-01" including inter- and
         extrapolated speeds (7800 and 8800 rpm); the calculation of Discharge
         Pressure ; Pressure Ratio; Adiabatic Head; Gas Power; Power-Loss;
         Required (total) Shaft Power; images of the worksheet Part 2 and Part 3
         are shown.
of this post is given here: link

1) Describing the principles behind the performance curves: a summary

We have seen earlier that the performance of a compressor is often presented by the manufacturer in a graphical way through the 'mapping' of how the pressure ratio of discharge over suction absolute pressure varies with volumetric suction flow rate for a number of compressor speeds. Alternatively the absolute discharge pressure for a given suction pressure can be 'mapped' for various speeds. In addition the shaft power required to run the compressor under the conditions shown on the map is given. See earlier Charts for the particular existing compressor "vscCR-01" under study.

A simulation of the performance of a compressor in an excel spreadsheet is not and should not simply be a mathematical formula(s) just capable of capturing these curves on the map through curve-fitting, regression etc. Such formula's are 'blind', are devoid of 'insight' into the essential principles at work in a running compressor and hence 'risky' to use outside the strictly specified operating conditions and parameters for which the compressor performance map was drawn. Parameters such as speed of rotation, feed rate range, gas quality with its many included physical property factors etc.

For this reason we have in the previous part discussed the principles of a centrifugal compressor and its essential characteristics:

> ===> a centrifugal compressor in essence *is a <u>volume</u> moving* machine where the rotor design (impeller diameter, width , shape), in conjunction with gas quality, determines the gas volume moved per rotation.
> ===> a centrifugal compressor *is a <u>head</u> producing* machine as the turning rotor causes the gas to exit the impeller's perimeter at high velocities creating   "velocity head" that in the static part of the machine is converted into "pressure head". (you could say "Bernoulli in operation")

<u>The volume of gas 'moved'</u> by an impeller is determined by the fixed geometry of the impeller (diameter, width of 'flow-channels', number and shape of blades), the number of rotations per unit of time, the nature of the gas and how this gas flows, its 'flow-pattern', through the impeller.

$$Q_1 \approx f \cdot Q_{imp} \cdot S$$

in which 'Q_{imp}' is the volume the impeller can move per rotation which is fixed as per the geometric design of the impeller, 'S' is the speed of rotation, 'f' is the factor characterizing the flow pattern and 'Q_1' is resulting the suction volume flow rate.

and thus we can write :

$$\frac{Q_1}{Q_{imp} \cdot S} \approx f = \sim \text{constant}$$

Note :   Let us check the SI dimensions of the expression :
Q1 (m3/s) = f (--) * Qimp (m3/rev) * S (rev/s) : ' f ' is dimensionless.
Other units are commonly used to obtain more easily 'handled' numbers like rpm. like Q1 in m3/hour and 'S' in rev/min or rpm. When these are used in this equation it means having to introduce additional constants to keep the equation whole and numerically consistent!

The 'head' producing capability of the compressor also roots in the geometric/mechanical design of the rotor and stator, the impeller(s) in particular. We have chosen to model the compressor as performing an *adiabatic compression* of a *real* gas. The impeller produces an adiabatic head 'H*imp*' (expressed in kJoule/kg)

$$H_{imp} \approx 4 \cdot \pi^2 \cdot S^2 \cdot D^2$$ in which 'S' is the speed of rotation, and 'D' the impeller's

diameter. The volume moving capability is proportional to the speed of rotation, while the head is proportional to the square of the speed ! Higher speeds benefits higher heads stronger.

The total adiabatic head 'H*ad*' produced by the machine (impeller+diffuser) is influenced by the *overall flow pattern* through the machine (combined rotor and stator parts):

$$H_{ad} \approx hf \cdot H_{imp}$$ in which '*hf*' is the factor characterizing how the overall flow pattern

is affecting the 'production' of adiabatic head 'H*ad*' generated by the machine.

$$H_{ad} \approx hf \cdot 4 \cdot \pi^2 \cdot S^2 \cdot D^2 \quad \text{and thus} \quad \frac{H_{ad}}{4 \cdot \pi^2 \cdot S^2 \cdot D^2} \approx hf\sim \text{constant}$$

Note: Let us check the SI dimensions :
Had (kJ/kg) = *hf* (--) * 4 pi * S^2 (rev/s)^2 * D^2 (m2) remember that
1 Joule=1Nm= 1kgm/s^2 * 1m then: Had (m2/s^2) = *hf* (--) * 4 pi * (1 /s) ^2 * m2
'*hf*' is dimensionless.

The unique performance of the existing machine is determined by its own geometrical configuration, its mechanical construction and its unique flow patterns established that are characterized by factors '*f*' and '*hf*'.

These equations suggest that in order to describe the "head producing" and "volume moving" capability we would need the construction details of impeller's, rotor, stator and more to analyze the flow patterns in order to asses how they affect those two capabilities, i.e. their the performance curves and power consumption!
However, we do not have such construction details that manufacturers are very reticent to share. But wait ! At this point it's good to keep in mind that we want to model the performance of our existing compressor, and do not want to step in the compressor designer's shoes! We want to model the machine from a user's perspective; we are looking from the outside in, and only want the variables we have control over to be (mathematically) described in terms of *the basic principles* at work in the operation of the compressor machine.

Therefore I have defined the following two variables to describe the performance curves of the existing compressor (see previous post Part I):

$$Q^* = \frac{Q_1}{S} \cdot \varepsilon_h \quad \text{and} \quad H^* = \frac{H_{ad}}{(S/1000)^2}$$

in which Q* (say Q-star) and H* (H-star) are *reduced* variables with respect to the rotation speed ; 'Q1' is the suction volumetric flow rate ; 'S' the speed of rotation ; 'Had' is the adiabatic head developed at speed S and flow rate Q1 and '*eps-h*' is defined as :

$$\varepsilon_h = Sqrt\left(\frac{2}{\left(1 + \dfrac{P_2 \cdot Z_1 \cdot T_1}{P_1 \cdot Z_2 \cdot T_2}\right)}\right)$$

in which 'P2' is the compressor's discharge pressure, 'P1' the suction absolute pressure, 'Z1' the compressibility factor of the gas at suction side, 'Z2' the compressibility factor of gas at discharge conditions and T1 and T2 the corresponding absolute temperatures. The Z factors, the gas compressibility factors, are calculated with the 'Zv' correlation published in earlier posts.

The ratio Q1/S, you could say, is the effective volume of inlet gas moved per rotation and 'eps-h' is the factor relating *inlet gas* density to the *average* gas density between inlet and outlet. The latter you could say accounts for the effect of speed on the average pressure across the machine and hence the average density of gas inside the machine!

The newly defined variable ' H* ' is calculated from the adiabatic head 'Had' that is in it's turn is calculated with:

$$H_{ad} = Z_1 \cdot \frac{R}{MW} \cdot T_1 \cdot \frac{k}{k-1} \cdot \left[\left(\frac{P_2}{P_1}\right)^{\left(\frac{k-1}{k}\right)} - 1\right]$$

The adiabatic Gas Power follows from: $G_{ad} = F_w \cdot H_{ad}$

Please note that the calculation of the adiabatic head 'Had' in the above formula used in the simulation model includes gas compressibility factor Z1 based on the inlet condition conditions and *not based on the average of in and outlet conditions*! For more on *how these definitions were arrived at* and used to make a "finger print" of our compressor machine see previous post with Part I.

2) Fingerprinting the Characteristic Performance (Model identification)

The fingerprint is the relationship describing the machine's essential performance in terms of the reduced variables H* and Q*. By plotting all available performance data in terms of H* and Q* I have found that the three performance curves (speeds 5870, 7000 and 8330 rev/min) have coalesced into *one single curve*. This single relationship between H* and Q*, unique to our machine, has been captured by regression in a quadratic relationship between H* and Q*. It is this curve that defines the unique, fundamental relation between H* and Q*. This relationship we have called the 'fingerprint' with as symbol 'H*fp'. This curve, as shown in the previous post, can be captured by a quadratic equation:

   H*fp = A * (Q*)^2 + B * (Q*) + C   for   0.258 =< Q* <= 0.438

With the help of this equation the compressor's H –Q curves can be simulated and performance map's generated also for interpolated or extrapolated speeds.

In the previous post we drew attention to the fact that all three surge points of the three original performance curves have now been "rolled" into *one single point* on the unified H*fp curve and that the (original) surge points and line can be reconstructed again from this single point by doing the reduction 'transformation' in reverse. In this Part II more details are is given in the following sections on the surge point and the choke point and lines, including the equations for the reverse "unpacking" of the *one single points on the H*fp curve* for surge and choke. For more details on the 'fingerprinting' see Part I.

3) Describing the Surge Points and Surge Line

No explicit information on the surge points or surge line in the performance map had been provided by the manufacturer. In the previous post, in Part I, I had assumed that the first point (lowest volume flow, highest pressure) of each of the three curves provided by the manufacturer was equal- or close to the surge point. In this Part II, I want to look closer into this. The H*fp variation with Q* has the shape of an inverted parabola (A is negative). The operating points with lowest flow rate on the curve are close to the "top" of the parabola while the slope of the curve gets increasingly smaller, the angle of the tangent line approaching zero degrees at the maximum of the curve.
Therefore I am defining the surge point as the point where the H*fp curve is at its maximum i.e. where the slope equals zero. The slope of the H*fp curve is:

$$d (H*fp)/d(Q*) = 2 A * (Q*) + B \quad \text{hence at surge: } 0 = 2A * (Q*surge) + B$$

$$===> \quad Q*surge = - B/ 2A \quad \text{and} \quad H*fp\text{-surge} = - B^2 / 4A + C$$

These are the coordinates of the (reduced) surge point on the finger print performance curve. From these coordinates the surge points of any speed curve can be reconstructed as follows: first we need to calculate what the pressure ratio P2/P1 is under surge conditions for a given speed of rotation 'S' as follows :

(a) Had,surge = H*fp,surge * (S/1000)^2 $\qquad H_{ad,surge} = H^*_{fp,surge} \cdot (S/1000)^2$

(b)   (P2/P1),surge = [Had,surge * MW / (Z1 * R * T1 * k/(k-1) + 1] ^ (k/(k-1))

Note that the other points of the curve are calculated directly the H*fp fingerprint equation. The actual suction volumetric flow 'Q1' at surge can be calculated from :

(c) Q1,surge = (Q*,surge) * S * 1/eps,surge

having the calculated (P2/P1),surge value, now the factor 1/eps,surge can be calculated as:

(d)   1/eps,*surge* = (½ + ½ * (P2/P1)^(1/k) * Z1/ Z2)^0.5

in which 'Z2' can be calculated iteratively (simple substitution) as is done for the simulation calculations of other points on the curve. Note that the last equation (d) includes already the adiabatic temperature 'T2' calculation.

These calculations (a) to (d) have been implemented in the upgraded Centrifugal Compressor Simulation spread sheet. In Section 10 a copy of the worksheet Part 2 (B)-2 is shown.

Note Having defined the surge point and determined its coordinates on the compressor's "fingerprint" performance map (plotted in reduced variables H* and Q*)
we can now describe the H*fp quadratic equation in an alternative way using these coordinates:

$$H*fp = H*surge – (A") * (Q* - Q*surge)^2$$

in which A" = -1 * A   or inserting the numerical values of the surge coordinates:

$$H*fp = 0.92760 – 7.5914 * (Q* - 0.24)^2$$

4) Describing the Choke Points and Choke Line

No explicit information on the choke points or choke line was divulged by the manufacturer either! What do we know about 'C*hoke*' ? The word itself already is very suggestive, you choke when you are overwhelmed by food or drink and cannot swallow it any more. That happens to a centrifugal compressor too when at a given speed and operating point (discharge pressure, volume flow rate) you start to increase the feed rate to the machine more and more. As the feed rate increases the outlet pressure drops (following the curve). At a certain high feed rate the outlet pressure starts to drop more steeply while the ability to process the increases in feed rate diminishes strongly until the outlet pressure falls dramatically low while the feed rate cannot be more increased: it appears as if the operation has run into *a stone wall*.

Why does this happen? An impeller increases the velocities of the gas flowing through it by the centrifugal forces acting on it. The velocities of the gas reach their highest value at the exit perimeter rim of the impeller. The compressor is designed for a given operating point of speed, feed rate and discharge pressure with the rotor and stator geometrically shaped to have their maximum energetic efficiency point, or in other words to have a minimum of energetic losses at that design operating speed and for that given gas quality. By more and more increasing the feed rate the internal gas velocities across the impellers consequently increase more too! The velocity increase is more prominent as the operating point moves along the curve towards lower outlet pressures and higher temperatures. When a point along the flow path inside the machine has reached a local gas velocity (likely the exit of one of the impellers) approaching the speed of sound of the particular gas, the "sound barrier" is reached. The velocity of sound cannot be exceeded! The choking point has been reached!

> (as an aside here: it is interesting to note how tech- people view and see their equipment / devices they are working with: they ascribe 'life' to it, see it like a living thing! it can make surging, regurgitating sounds ; it can choke on what it is fed etc   perhaps you can find more examples of similar sayings ?)

To evaluate where our compressor is with respect to the choke points or choke line we would need to look at the internal velocities, however without detailed info of the internal dimensions that is impossible. We can however calculate what the discharge *internal volumetric flow rate Q2* is for a given operating point on the curve. Our model allows us to calculate the gas density Rho2 at discharge Pressure P2 and hence:

$$Q2 = Fmass/ Rho2 = Q1 * Rho1 /Rho2$$

If we calculate Q2 for all three curves provided by the manufacturer we can see an interesting picture emerge: for each of the three curves *the first point* indicates that Q2 equals about 0.25 m3/s, then for the next points along the curve Q2 gradually increases while the last points provided show the volume flow rates of the compressed gas Q2 are approaching a value of about 0.5 m3/s irrespective of speed and discharge pressure (P2) level generated. To be precise *the last point* of the speed curve at 7000 rev/min we see Q2 reaches 0.50 m3/s ; for 8330 rev/min Q2 reaches 0.56 m3/s while for 5870 rev/min the last point (of only five) reaches 0.43 m3/s.

These observations all hint, in my mind, to the conclusion that all the last points on the manufacturer's curves are approaching a limiting value for the Q2 volume flow rate. Without detailed construction data we cannot ascertain or estimate the highest velocities values, we can however make an order of magnitude check using the apparently converging Q2 flow rate value of 0.56 m3/s and the fact that the speed of sound in propane at 20 Bar is around 250 m/s (see later post for this).

The last points provided by the manufacturer likely (I am speculating) are the 'last' points closest to their choke points that can be safely operated. Therefore for our modeling purposes I am <u>defining the Choke point</u> as the last point on the H*fp curve at which point the Q* value equals 0.440 (rounded value).

From this defining value of Q*,choke =0.440   we find   H*fp ,choke = 0.62389 . This value simply follows from calculation with the fingerprint equation H*fp (section 2).

These two values are the coordinates of the (reduced) *Choke Point*. From these coordinates the choke points of any speed curve can be reconstructed (and hence the *Choke Line*) in the same way as described in Section 3 for the Surge Point and Line with equations (a) to (d).

5) **Notes on the Shape and Slope of the "Finger Print" H*fp Curve**

In the above discussion of the surge point and the choke point I have referred to the shape and the slope of the performance curves and in particular to the essential, reduced performance curve that I have called the "fingerprint" curve "H*fp ". For example we have defined *the surge point* as the point where the H*fp curve reaches its maximum value i.e. when the slope of the tangent line to the curve approaches a value of zero, i.e. the tangent line running parallel to the Q* axis (its angle = 0). It is interesting to note that the slope and angle of the tangent line in the (manufacture's given) first point on the curve has an angle of minus 16 degrees. The initial part of the curve between the newly defined surge point and this first point is hence rather 'flat'. From a control point of view it means that a small change in head H* is associated with a much larger change in Q* value and therefore small disturbances in head caused by small, frequent variations in rotation, pressures, temperatures, gas quality – process and equipment noise – cause much larger changes in Q* and hence the compressor operating in this part of the curve cannot "find" and "settle" on a single, fixed operating point causing instabilities in the operation and these maybe the precursors to surging conditions. (see Chart given below).

In the discussion of the choke point we referred to the fact that when this point is reached during the operation of a 'campaign' (or a test) of 'pushing' the feed rate as high as possible, the outlet pressure drops, the performance curve sharply bends into a line at a 90 degree angle to the flow axis (the stone wall).

It is interesting therefore to look a bit closer at the shape and slope of the performance curve. The performance maps as presented in previous Charts of Part I can be somewhat deceiving as they look rather "flat" because of the different scales at which they are plotted! Let us examine how the slope of the "fingerprint " curve H*fp varies with Q* . The 'slope' = 2A * (Q*) + B   and the angle of the tangent line in degrees = 180/pi * ATAN (slope).

In the following Chart I have plotted the H*fp equation versus Q* at equal scales so as to have a proper impression of the steepness of the performance curve. In addition also the Surge - and Choke points (as defined) are shown together with a number points marked with the angles of their tangent lines.
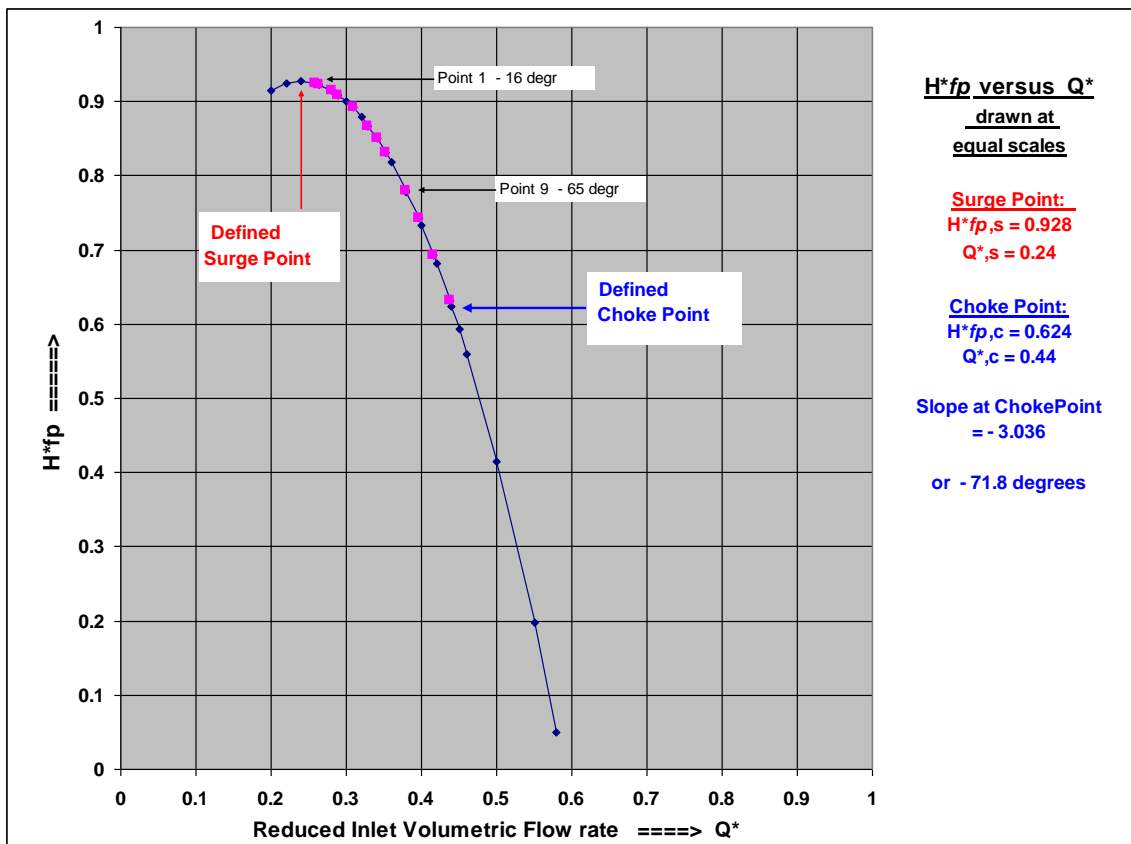
Diagram-1 Fingerprint H*fp versus reduced Inlet Volumetric Flow rate Q* drawn at equal scales.

The Diagram shows for three of the 12 points the angles of their tangent lines, whereas on the right hand side of the Chart the coordinates of surge – and choke points are given. Note that the performance curve at the choke point is already sloped downwards at angle of nearly -72 degrees! That fact also has implications for controlling the compressor operation. Is the initial part of the curve close to the surge point nearly 'flat', for the end part we find the curve gets rather steep where small variations (this time) in Q* have a much larger effect on H* and hence instabilities here also play a larger role in the operation.
Note also : To study a potential change in the whole suction and discharge piping system of which the compressor is part, for example to save on energy consumption by the compressor, the "system pressure loss- or resistance curve" needs to be drawn and

examined in this performance diagram. In particular it is the intersection of these two curves that determine the operating point of the compressor. It is the angle between these two curves that needs to be looked at vis a vis the controllability.

6)   The GasPower 'Gad' and the unified G*ad 'fingerprint' curve

The Adiabatic Gas Power required to perform the adiabatic compression of the gas in our machine follows straight forwardly from the model calculations as already summarized in Section 1 and further detailed in Part I in the previous post!

The Adiabatic Gas Power "Gad" at a given operating point on the curve follows directly from the adiabatic compression calculations: it is simply the product of the mass flow rate and the adiabatic head 'Had'

   Gad = Had * Fm   (kW = kJ / kg * kg / s)

We can express this relation also in terms of the two new variables, the reduced head H* and the reduced inlet volume flow rate Q* that we introduced to find the fingerprint relation, the essential relation between the adiabatic head Had and the inlet volume flow Q1 for a given speed of our existing compressor machine under study. The head – flow curves for the three given speeds when expressed in terms H* and Q* have now 'coalesced' to form *one single* curve, the finger print Head curve that is independent of speed. This curve can be regressed in a quadratic equation that we have labeled : H*fp *the finger print equation.* Remembering that we defined as H* = Had / (S/1000)^2 and that the mass flow is Fm = Rho1 * Q1   and that Q* = Q1 / S * eps-h   we can write:

   Gad = (H*) * (S/1000)^2 * Rho1 * (Q*) * S / eps-h

Note that in our definitions of H* and Q* we have kept the following units:
Had in kJ/kg , Q1 in m3/hour and S in rev/min and Rho1 in kg/m3 in order to obtain values for H* and Q* between 0 and 1 .
Hence to keep the units of the right hand side consistent with the left hand side (kWatt) and to avoid dealing with large numbers like S^3 additional constants have to be introduced and after rearranging the expression becomes:
   Gad = (H*) * (Q*) * (S/1000)^3 * 1000/3600 * Rho1 /eps-h

By Defining G*ad (say Gad-star) as:

G*ad = Gad / (S/1000)^3 /Rho1 * 3.6 * eps-h

We can simply write the relationship between adiabatic gas power, the head and flow in terms of the three reduced variables in a form analogous to the original equation directly following from the detailed adiabatic model calculations:

         G*ad = (H*) * (Q*)

Now you may say that is a fine algebraic manipulation of variables and so on but so what? In answer to that: the advantage of this formulation is that we can now directly relate the adiabatic gas power at a given operating point with the finger print relation H*fp between H* and Q* and thus:

G*ad = H*fp * (Q*)   or   G*ad = (A*(Q*)^2 + B*(Q*) + C) * (Q*)

From this we learn that Gad is a function of the *third power of the speed* and the *third power in Q** (as polynomial) as well !
We have seen that the H*fp expression alternatively can be written as:

H*fp = H*surge – (A") * (Q* - Q*surge)^2

In which H*s and Q*s are the coordinates of the surge point in the H* - Q* diagram.
With this alternate H*fp expression we can also write this relation in an equivalent but more insightful form:

G*ad = [H*s – A" * (Q*-Q*s)^2] * (Q*)

With these relations between the three reduced variables, G*ad, H* and Q* we have extended the H*fp equation into the reduced adiabatic Gas Power relationship. The H*fp equation is based on a *regression correlation* of H* - Q* values obtained from the adiabatic compression calculations (see 'Part I' in previous post). Hence the G*ad relation is the *unique fingerprint of the adiabatic gas power requirement* of our machine to obtain the achieved compression performance.

The values of Gad follow directly from the Had and Fm values generated by the model and thus if we plot the G*ad relation versus Q* and co-plot the directly calculated Gad values for all three speeds for all 25 points and they should all fall at - or very close to the G*ad fingerprint curve ! See next Diagram.

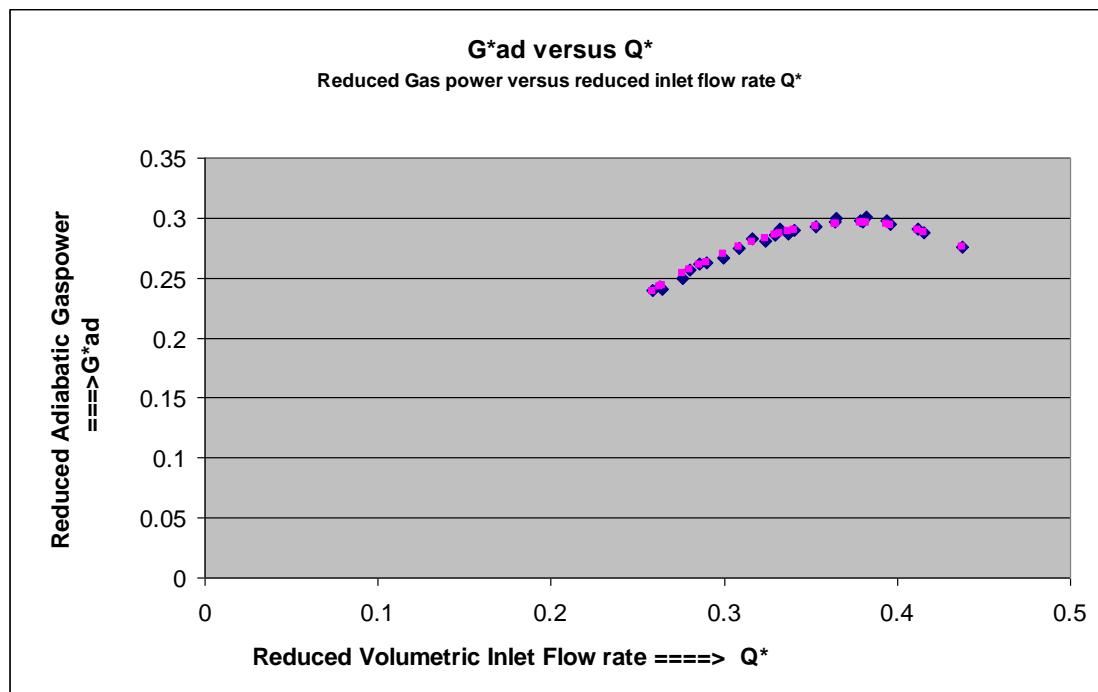Diagram-2 The reduced Adiabatic Gas Power " G*ad " versus the reduced Inlet Volumetric Flow rate Q*.

To summarize : in this section 6 I have extended the adiabatic compression sub-model by defining the *reduced* Gas power G*ad. This reduced variable relates directly to the reduced Head 'H*' and the reduced flow rate 'Q*'.

7) The Shaft Power "Ps" and the Adiabatic Efficiency Factor "Eta-ad"

The shaft power to drive the compressor is in part consumed by the production of increased outlet pressures, being the adiabatic gas power Gad part, and the remainder absorbed by "energetic losses" incurred through concurrent loss processes.
Where we have a theory to describe the adiabatic gas power part, as used in our model, there is no straightforward single piece of theory to describe the "loss" part.
There are many contributors adding to - and factoring into these energetic losses in their totality. I will come back to these in the next section 8-1.

The Gas Power required to perform the adiabatic compression of the flowing gas in our machine follows straight forwardly from the model calculations as already summarized in Section 1 and further detailed in Part I in the previous post and further "extended" in section 6 of this PART II.

In our earlier analysis of the existing compressor's performance as given in Part I we have related our theoretical calculation of 'Gad' to the actual required shaft power by means of the Adiabatic Efficiency Factor "Eta-ad". Once this factor is known the shaft power can be calculated as:

  Ps = Gad / Eta-ad.   (units: kW = kW / (--))

'Eta-ad' is here the sole factor, the key to arriving at the *total* required shaft power to drive the compressor. This factor in one single number wraps the gas power and all the contributing flow resistances and their corresponding power losses into a single number!

No single piece of theory exists, as far as I am aware of, to predict the extent of the sum-total of these losses, although over time empirical or semi-empirical correlations of contributing factors have been developed to be able to estimate a part of these losses. The capability to predict such losses is important in particular from the compressor designer's perspective.

Therefore through 'looking' at the model's data and the manufacturers shaft power numbers, through trial and error I found a correlation for the adiabatic efficiency factor 'Eta-ad' and how it relates to the flow variable I had defined as 'Q* ' as follows:

Eta-ad * (S/1000)^0.39 / (Q1/S) = A * (Q*) + B   …………………………(eq.7-1)

in which the constants are: A= -8.4625 and B = 6.082

This correlation allows the total shaft power 'Ps' to be calculated from 'Gad' with an average percentage error of 0.67%!! Not bad! See Part I showing a Diagram where the predicted shaft power and the manufacturers quoted shaft power numbers are co-plotted to compare them in a visual way.

This "powerful" correlation I found, remarkable though it is, is only strictly valid for this compressor machine, the vscCR-01, to use as predictor! It is a correlation of connected

variables that influence each other, however it does not reveal much insight into the scope and limitations of the application- and use of this equation. Therefore if you are going to apply this correlation in the simulation of your compressor beware and be cautious.

It is for this very reason that in this PART II, that I have taken a closer look, attempting *to find other ways* to calculate the total shaft power requirement 'Ps' by focusing directly on the power losses themselves in order to get a better 'feel' and more insight through developing a relationship for these losses with the operating parameters and variables of the compressor. This is dealt with in the next section 8.

8) **Fingerprinting the Overall Power – Loss Process(es).**

8**.1 Determination of the losses during operation of compressor "vscCR-01" .**

Energetic losses in a compressor are due to 'flow resistances' present along the path-way a gas follows through the machine. Both the stator and the rotor part contribute to the losses. The impeller, it's geometric shaping and dimensioning, plays a major role in the generation of losses. Study and research over the decades have identified various types of resistances contributing to the overall loss of power experienced. Losses such as "incidence loss", impeller "skin friction losses" and many more! Such detailed account of loss factors and how to describe these and how to calculate these in detail are of prominent importance to compressor designers and manufacturers, witness the large amount of studies published devoted to this subject.
From our perspective the question is can we find a description of how the *overall, the sum-total of all these loss factors*, relates to the adiabatic gas power production in vscCR-1 and how it figures into the total shaft power requirement to drive the compressor.

In PART I we have described the approach taken to formulate a Model of this variable speed compressor vscCR01. The Simulation Model conceptualizes the variable speed compressor, shown in the block diagram 4 of Part I, as consisting of two processes occurring side by side: the adiabatic compression process and a "loss process". This implies that the total shaft power is *the sum* of the gas power plus the power consumption by the 'loss processes. The 'Power Loss' part hence can be directly calculated from the Manufacturer's quoted shaft power 'Ps' (in kW) and the adiabatic gas power 'Gad' calculated with our adiabatic compression sub-model:

$$Ps,loss = Ps - Gad \quad \text{(all expressed in kW)} \quad \ldots\ldots\ldots\ldots\ldots.(eq.8\text{-}1)$$

Let us picture how these losses look by calculating for each of the three speeds the power losses and plot these against the suction volumetric flow rate: (click on picture to enlarge):

Power Loss 'Ps, loss' versus 'Q1' flowrate for "vscCR-01" for three compressor speeds 5870, 7000 and 8330 rev/min

Ps,loss = Ps - Gad

8330 rev/min

7000 rev/min

5870 rev/min

Ps,loss ====> kW

Inlet volumetric Flow rate 'Q1' ====> (m3 /h)
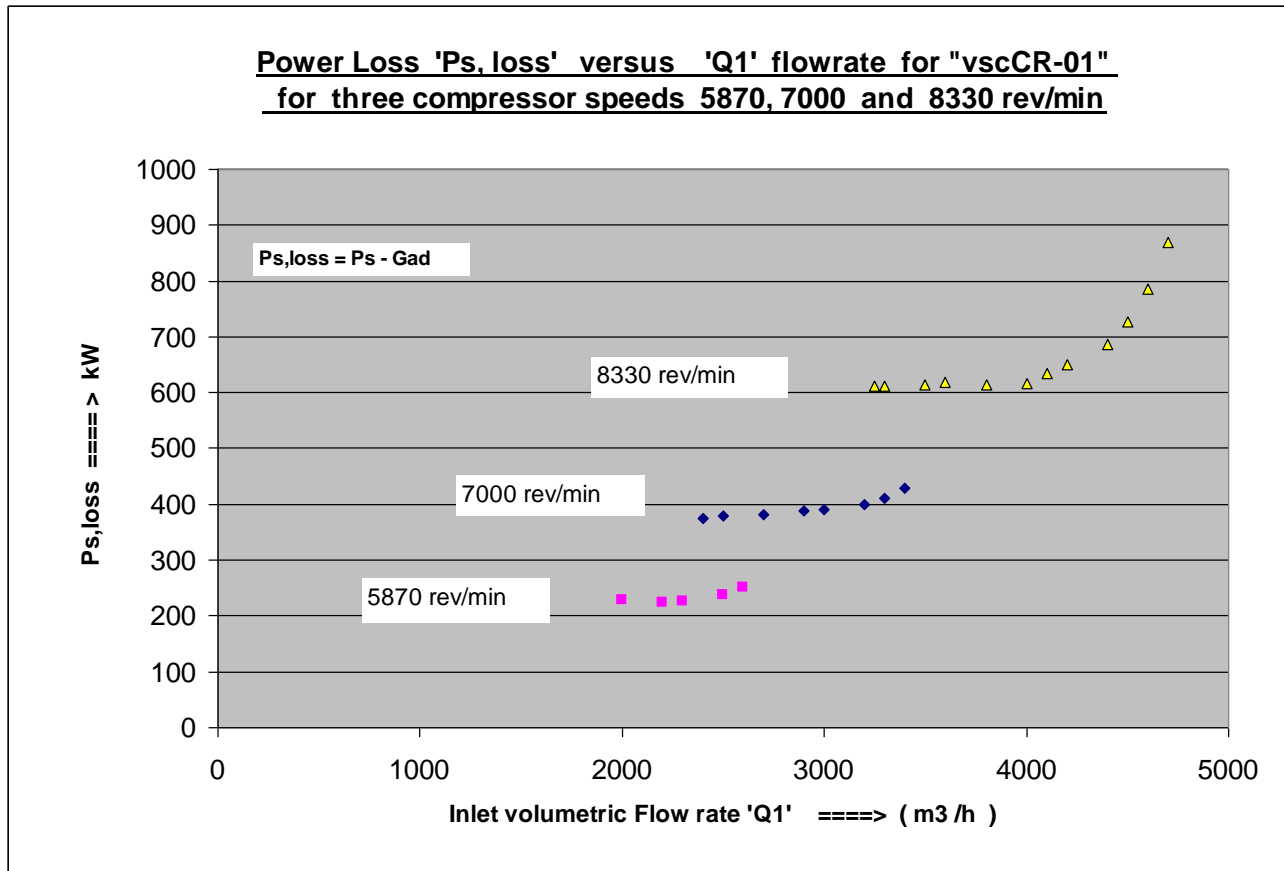
Diagram-3 The Power Loss 'Ps,loss' versus inlet Volumetric Flow rate 'Q1'

The graph shows the higher the inlet flow rate the steeper the rise in 'Ps,loss' i.e. in the part of the (shaft) power part that is *not converted* into a pressure rise! The highest speed curve shows that rather dramatically! One thing worth noting in the graph is that the first part of each loss curve is rather flat! I will come back to the significance of this observation in a future post (effect of combined incidence and friction losses)!

Next let us focus on the 8330 rev/min speed curve in detail and see how it's (total) shaft power, adiabatic gas power and the power loss behave when shown in one single graph (click on picture to enlarge):

**Compressor Shaft Power , adiabatic Gas Power and Power loss
for compressor "vscCR-01" at 8330 rev/min versus Q1 flowrate**
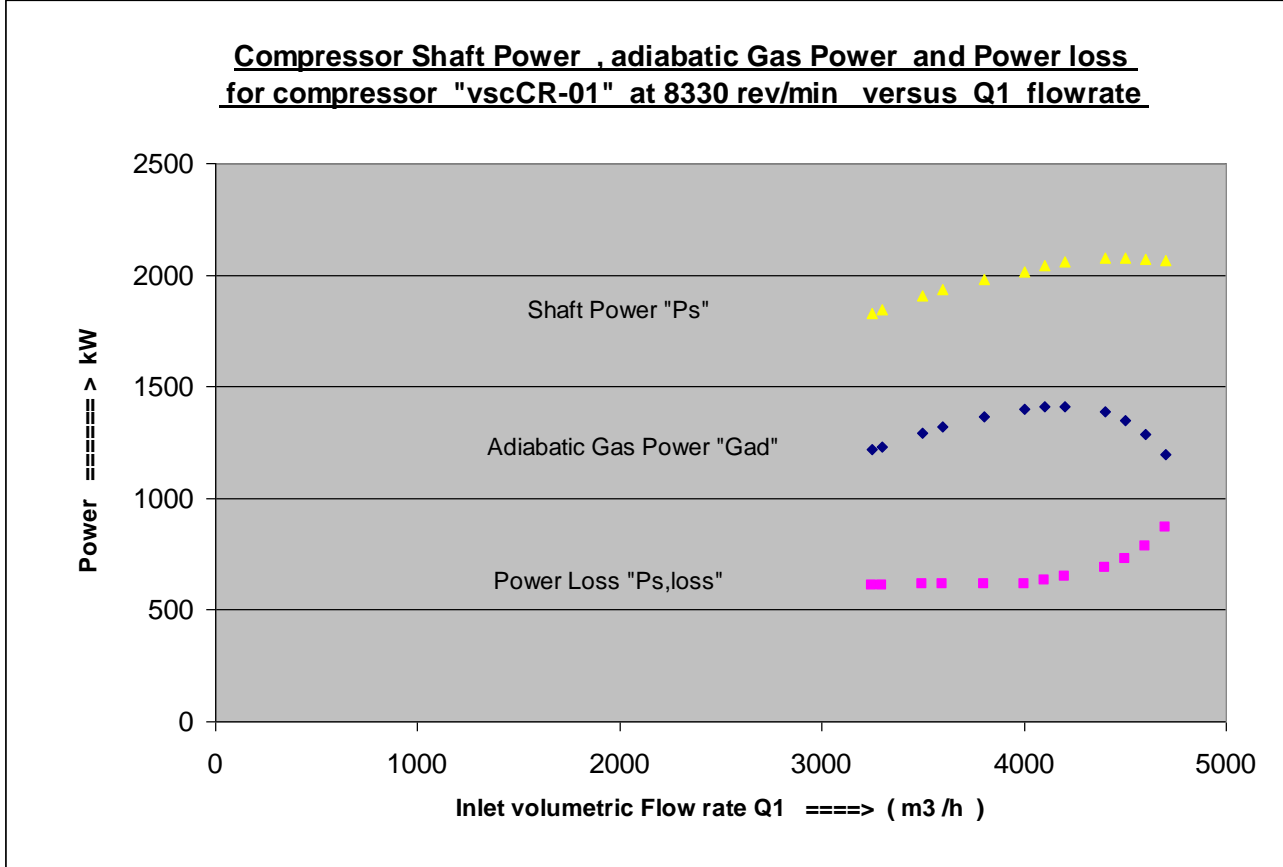
Diagram-4 Compressor Shaft Power , Adiabatic Gas Power and Power Loss versus Inlet Volumetric Flow rate 'Q1'.

The total shaft power, not unexpectedly, increases with increasing volume flow rate. The curves for the two concurrent processes: the adiabatic compression and the power loss process(es) show a very interesting pattern their curves look like they are each other's mirror image! And that is no coincidence as I will show later. Similar pictures are found for the two other speeds.

' **8.2 Pressure Loss and Power Loss**

Therefore let us examine a bit closer the losses at a constant speed of rotation. Power added to the shaft that is "lost" towards the ultimate purpose of the machine to raise the pressure of the gas exit stream, is a loss of pressure (increase) not achieved. The term "pressure losses" is a familiar one in the context of hydraulic flow systems and engineering science. Thus let us start with the most well known and used semi-empirical relation to describe pressure losses across a 'conduit' carrying a fluid flow to aid in analyzing the "overall" losses:

Delta P = K * ½ * Rho * v ^2   …………………………………………..(eq.8-2)

In which 'Delta P' is the pressure difference along the conduit and in which "K" is a factor characterizing the *total resistance to flow* through the conduit. 'Rho' is the fluid density and 'v' the average velocity. (Note f = Fanning friction factor and 4f the Moody friction factor for pipe flow; see principles described in a future post).

In our case 'P2' and 'P1' are the compressor discharge and inlet pressures respectively; the density 'Rho' is not constant as we are dealing with a gas compressible flow and as we have elaborated in section 3.2 and 3.3 of PART I, equation eq.8-2 takes the following form:

Delta P = K * ½ * 1/ Rhoavg * (Fm/ Ac)^2

 or

P2 – P1 = DeltaP = K / Ac^2 * ½ * Q1^2 * Rho1^2 / Rhoavg

in which 'Ac' is the *average cross-sectional area* and 'Rho1' is the gas density at the inlet; 'Rhoavg' is the (algebraic) average density inside the machine over in- and outlet and 'Fm' the mass flow rate and Q1 is the inlet volumetric flow rate.
Note 1: the dimension of Pressure [P1] and [P2] = N/m^2 or Nm / m^3.
Note 2: the pressure difference here has a negative sign see comments in future post.

If we divide both sides by 'Rho1' the left hand side obtains the dimension of Nm /kg or Joule/kg and we get:

(P2 – P1) /Rho1 = K / Ac^2 * ½ * Q1^2 * Rho1 /Rhoavg

In PART I we have <u>defined the ratio</u> Rho1 /Rhoavg as "eps-h^2" and thus:

(P2-P1) /Rho1 = K / Ac^2 * ½ * (Q1*eps-h)^2 ………………….(eq.8-3)

The power required to drive the flow of fluid through the resistance formed by the shape and size of the conduit, we find by multiplying with the mass flow 'Fm' in kg/s

(P2-P1)/Rho1 * Fm = K/(Ac)^2 * ½ * (Q1* eps-h)^2 * Fm

The left hand side is the power (loss) in Watts.

By *analogy* we are now applying this relation to the *overall* power losses generated by the resistances posed to the gas flowing through the internals of the compressor or more precisely said*: while* being driven by the centrifugal forces via the diffuser to the discharge. Granted, the "conduit", the 'flow-channel' through which the gas is passing from inlet to discharge is of complex form and shape for sure! Nevertheless, we equate the overall power loss "Ps, loss" in the compressor with the right hand side of the equation:

Ps,loss = K / Ac^2 * ½ * (Q1*eps)^2 * Fm ……………………(eq.8-4)

Note1: Let us check dimensions when expressing in SI units:
Watt = J/sec = (--) / m2 * (m3/sec)^2 * kg/sec kgm/sec^2
Note2: "Ps" is the total shaft power to operate the compressor; Ps,loss the part of Ps consumed by overcoming resistances to flow and converted into heat.

The factor 'K' is the sum-total of all contributing resistance factors and is applied here analogous to the 'accounting' of resistances as is done in the 'theory and practice' of hydraulic systems, for instance liquids flowing through a piping system, consisting of straight long pipe fitted with a series of different "fittings" like pipe bends, pipe diameter

changes, valves, orifices and so on. The additional flow resistances that these fittings add to the total have been (semi-) empirically measured and determined and can be added to the value of 'K'. An interesting piece of information is the fact that in hydraulic systems the factor 'K' is *not constant,* even after the accounting for the configuration of the conduit carrying the flow, but is still dependent on the mass flow rate. For example for turbulent flow through *smooth long circular* pipes H. Blasius found that the "friction factor" depends on the Reynolds number as follows:

"friction factor" =   Constant / Re^1/4   for turbulent flow 4000 < Re < 10^5

Noteworthy is the *inverse relation ship* with flow rate as indicated by the inverse Reynolds number, meaning the higher the flow rate the lower the friction factor!
In the further development of equation 8-4 we should keep in mind that the factor 'K' itself may be a function of flow rate (volume or mass or reduced volume flow rate Q*).

'8.3 Characterizing the Overall Loss Process(es).

For centrifugal compressors the different resistances adding to the overall resistance to flow are many and complex and have been studied and used in the design of compressors in order to minimize energy losses at the required – design – operating point. As said we will keep to the user's perspective in our analysis not the designer's and try not 'zoom' too much into the *detailed* individual loss contributors but keep the focus on the overall losses.

The adiabatic compression theoretical part of the Compressor Simulation Model calculates the adiabatic head, equal to the difference in enthalpy of the gas between inlet and discharge, at constant entropy. The adiabatic head 'Had' is expressed, like enthalpy, in kiloJoule per kilogram of gas.
To analyze the loss process and fingerprint the essential, characteristic of the (overall) loss process we should therefore similarly look at the power loss per kg of gas!
Let us see how the last diagram looks if we now plot 'Had' together with 'Ps,loss' and 'Ps' also expressed on a 'kJ/kg' basis by calculation of 'Ps,loss' divided by the mass flow rate 'Fm' and do the same for 'Ps', all for a constant speed of 8330 rev/min versus mass flow rate as shown in the next Diagram (click on Chart to enlarge):
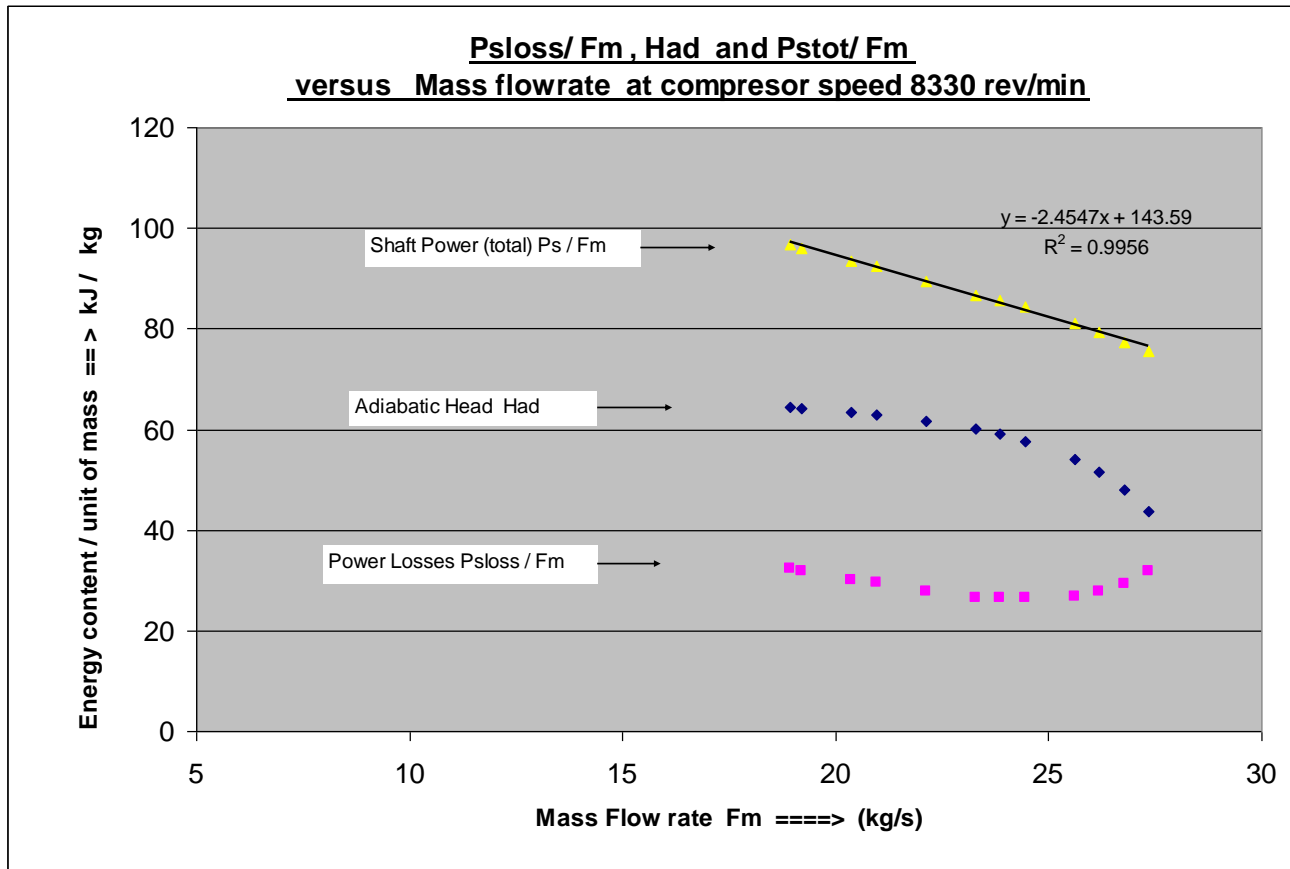
Diagram-5   'Ps,loss/ Fm' and 'Had' and 'Ps/ Fm' versus Mass Flow rate 'Fm'.

Looking at this graph immediately the steady, linear decrease of the shaft power divided by the mass flow rate jumps out! But just as striking is the fact that the "mirroring" of the adiabatic head curve with the curve of the power loss over mass flow rate has become even more pronounced and clearer!

'8.4 New reduced variable " H*,loss " + Fingerprinting the Overall Loss Process.

In the following we are searching, with the help of the last equation eq.8-4, for insight into how the losses relate to the operating conditions of flow, speed and pressure. Can we probe into these losses (without having construction details of internals) by analyzing the operating data in the same manner as done for the performance curves and adiabatic gas power modeling? And the answer is yes! How? By trying to find the unique relationship describing the losses, a unified, "fingerprint" relation for the overall resistance factor "K" that according to eq.8-4 lies at the core of the *overall* loss phenomena exhibited by this compressor.

Let's express the power loss in kJ/kg by taking eq.8-4 and dividing both sides by the mass flow rate 'Fm' and we get:

Ps,loss / Fm = K / Ac^2 * ½ * (Q1 * eps-h)^2

Note: the dimension of the left hand side: kW /(kg/s) = kJ/kg.

If in *analogy* with our definition of the reduced adiabatic head H* = Had /(S/1000)^2
(in kJ/kg/ (rev/min)^2) we again divide both sides but this time by (S/1000)^2 we get:

   Ps,loss / Fm / (S/1000)^2 = K / Ac^2 * ½ * (Q1 * eps-h)^2 / (S/1000)^2

and remembering and including the definition of Q* in PART I and rearranging we get :

   Ps,loss / Fm / (S/1000)^2 =1E+6 * K / Ac^2 * ½ * (Q*)^2 ……………..(eq.8-5)

On the left hand side we have the energetic losses expressed as a *new reduced
variable* with on the right hand side an expression involving the *reduced v*olume flow
rate Q*.
Therefore I am defining the left hand side of eq.8-5 as "H*loss" .

   H*loss = Ps,loss/Fm/(S/1000)^2 = 1E+6 * K / Ac^2 * ½ * (Q*)^2   ………(eq.8-6)

The question now is does this equation describe the power losses shown in the first
Diagram of 'Ps,loss' versus inlet volumetric flow rate Q1? If indeed the factor 'K' , the
overall resistance to flow, is a constant the answer should be yes, because we have
here the essential relationship of energetic losses with the square of the flow rate. This
means that if for all three speeds and all points we plot the left hand side, 'H*loss'
versus Q* we should get all data points to 'coalesce' into a single line!
Therefore I plotted the left hand side for all 25 data points against Q* and found, as you
can see in the next Diagram-6, the points show some 'aggregation' into a sort of band
but not a line as the next Diagram shows.

Ps,loss / Fm / (S/1000)^2   versus reduced volume flow   Q*
for all three speeds for compressor "vscCR-01"

$y = 41.3e^{-6.8933x}$
$R^2 = 0.9095$

$y = 168.79x^2 - 144.62x + 33.469$
$R^2 = 0.9162$

Total, Overall flow Resistances factor "K"
for 25 points (and 3 speeds)
plotted against reduced vol flowrate Q*

Ps,loss / Fm / (S/1000)^2   ====>

Q*   =====>
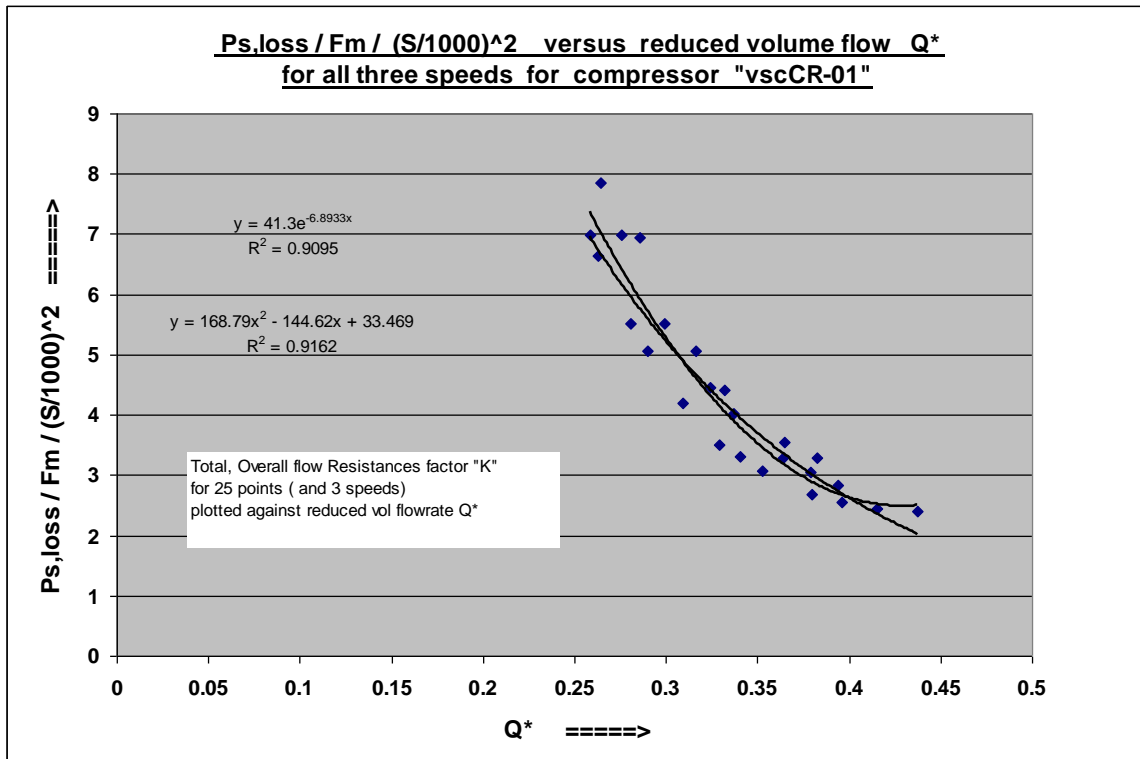
Diagram-6 'Ps,loss/Fm/(S/1000)^2' versus reduced Inlet Volume Flow rate 'Q* ' for 3 speeds.

The points nevertheless show a definite trend: the higher the reduced flow rate the lower the value of the resistance factor 'K'. When performing a regression on this scattered band only low r^2 values are obtained (see Diagram-6).
Next, then tried plotting the 'K' values versus the inlet volumetric flow 'Q1' and following this plot I made yet another plot against the mass flow rate. Both these plots (not shown here) displayed *three distinctly separate* lines for each of the speeds, with no aggregation, no banding, no coalescing of points into a single line, no unification. Now what to do?

Then remembering the possibility of 'K' not being a true constant! In addition Then furthermore remembering that even for a simple geometric shape like a straight circular pipe the overall resistance factor 'K' is *not constant* and still shows some dependency on mass flow rate. Therefore, with the "complex form and shape" of the gas flow channel (an understatement) present in our compressor we should check how 'K' in equation eq.8-6 behaves! So let us see if indeed 'K' varies with 'Q*' and if so how. In the next Diagram I am plotting 'K' versus Q*. With 'K' based on eq.8-6 as:

  K = H*loss /(Q*)^2 = Ps,loss / Fm / (S/1000)^2 / (Q*)^2 * 2E-6 * Ac^2 ….(eq.8-7)

Ignoring for the moment the factors 1E-6 * 2 * Ac^2 as truly constant then calculating the values of 'K' for all 25 data points (three speeds) and plotting these against 'Q*'. I found these points form a much narrower a band. Then next by trial and error I found that all data points could be *forming a single line* if plotted against the following expression: " 1/ (Q*) * eps-h^0.44 " as the following graph shows:

'K' factor = Ps,loss / Fm / (S/1000)^2 /Q*^2   versus   1/(Q*) * eps^0.44 for compressor "vscCR-01"

Y-axis: Ps,loss / Fm / (S/1000)^2 / Q*^2 =====>
X-axis: 1/ (Q*) * eps-h^0.44 =====>

$y = 0.3098e^{0.9591x}$
$R^2 = 0.995$

$y = 2.2731x^2 - 8.1128x + 9.4235$
$R^2 = 0.9987$

Total, Overall flow Resistances factor "K"
for 25 points (and 3 speeds)
plotted against inverse reduced vol flowrate
Q* times eps-h^0.44
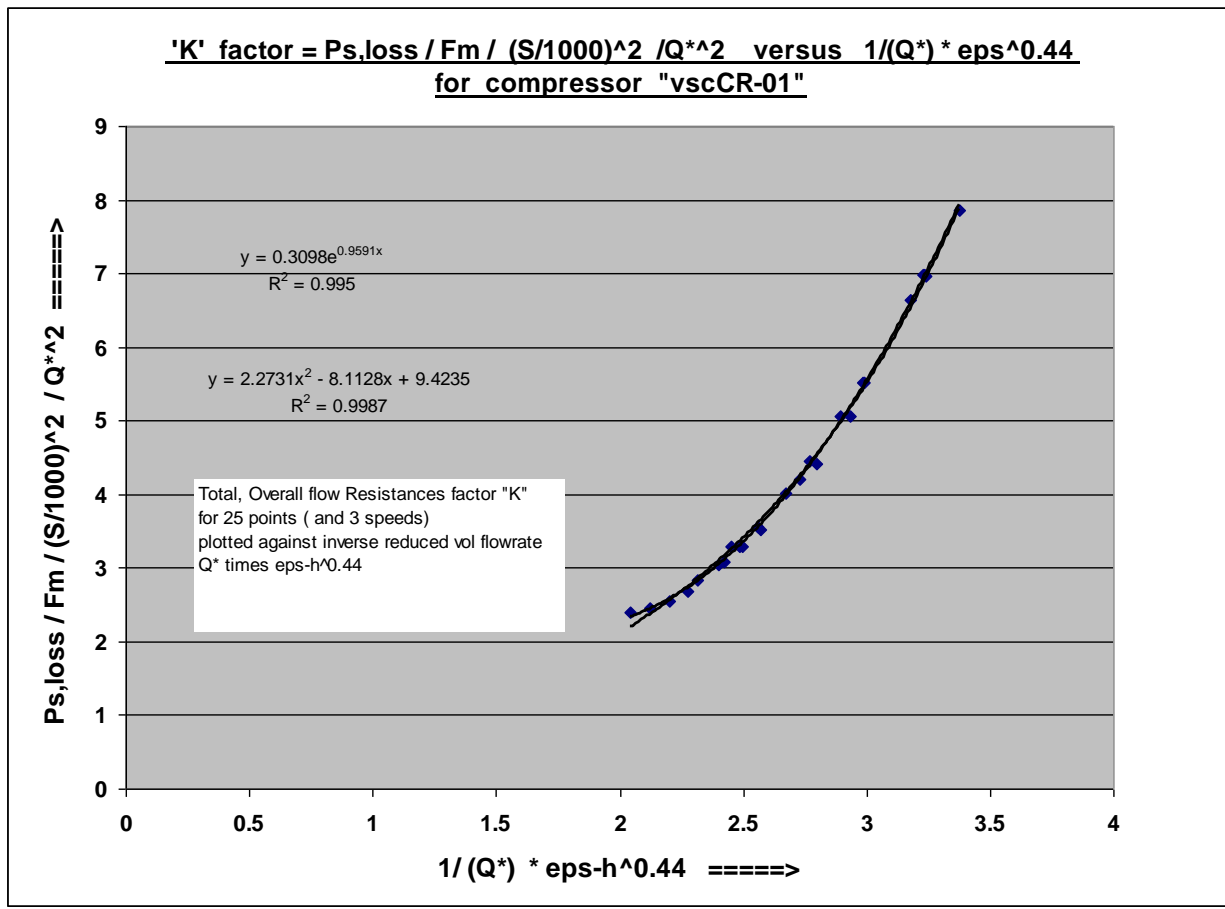
Diagram-7 Fingerprinting the flow resistance **K***fp for compressor vscCR-01.

This Diagram shows that the overall flow resistance factor 'K' characterizing compressor 'vscCR-01' is inversely related to the reduced flow rate Q^*. In other words the higher the flow-rate through the compressor the lower the overall resistance factor becomes. And as we have seen before, this is not unlike the relationship in hydraulic systems like the one quoted except here the inverse relationship is much more pronounced. As shown in the Diagram the resistance factor values plotted are very well described by the following quadratic equation:

  overall resistance factor **K** = A*x^2 + B*x +C ……………………………… (eq.8-8)
                in which x = 1/(Q*) * eps-h^0.44
                and A = 2.2731 ; B = -8.2188 ; C = 9.4235

The bold symbol **K** effectively is a re-definition of 'K' of eq.8-7 to include the factor A*c* being the inverse of the cross sectional area averaged over the flow path and the conversion factors related to the units employed in this equation: kilo Watts, mass-flow in kg/sec and speed in rev/min instead of Watt, kg/hr, rev/sec or radians/sec.

This quadratic relationship of **K** with Q^* is *the essential, overall resistance "fingerprint" function* describing how the overall resistance posed to the gas flowing through the internals of the compressor during the operation of 'vscCR01' varies with the reduced inlet volume flow rate. I will refer to this relation of **K** (Q^*) with the symbol "**K***fp " :

    K*fp = A * (eps-h^0.44 / Q*)^2 + B * (eps-h^0.44 / Q*) + C …… . eq.8-9

With the constants A, B and C as given before (eq.8-8).
The *reduced loss* 'H*loss' of eq.8-6 can be compactly written in reduced variables as:

    H*loss = (**K***fp) * (Q*)^2 ……………………………………..…..…(eq.8-10)

With the help of equation eq.8-9 the power loss 'Ps,loss' then is calculated as:

  Ps,loss = (**K***fp) * (Q*)^2 * Fm * (S/1000)^2   ……………………..…..(eq.8-11)

('Ps,loss' is in kW if 'Fm' is expressed in kg/sec and 'S' in rev/min).

With eq. 8-11 and the resistance fingerprint eq. 8-9 we should be able *to predict* the shaft power losses, i.e. that part of the shaft power *not* converted into a gas *pressure rise* but instead converted into a temperature rise !

'8.5 Predicting the PowerLoss 'Ps,loss' .

Taking the combination of equations 8-9 and 8-11 and calculating the values for Ps,loss as function of Q^* the following Diagram shows these predicted values that have been co-plotted with the power loss values directly calculated from the Manufacturer's quoted shaft power minus the Gas Power values. (click on chart to enlarge):
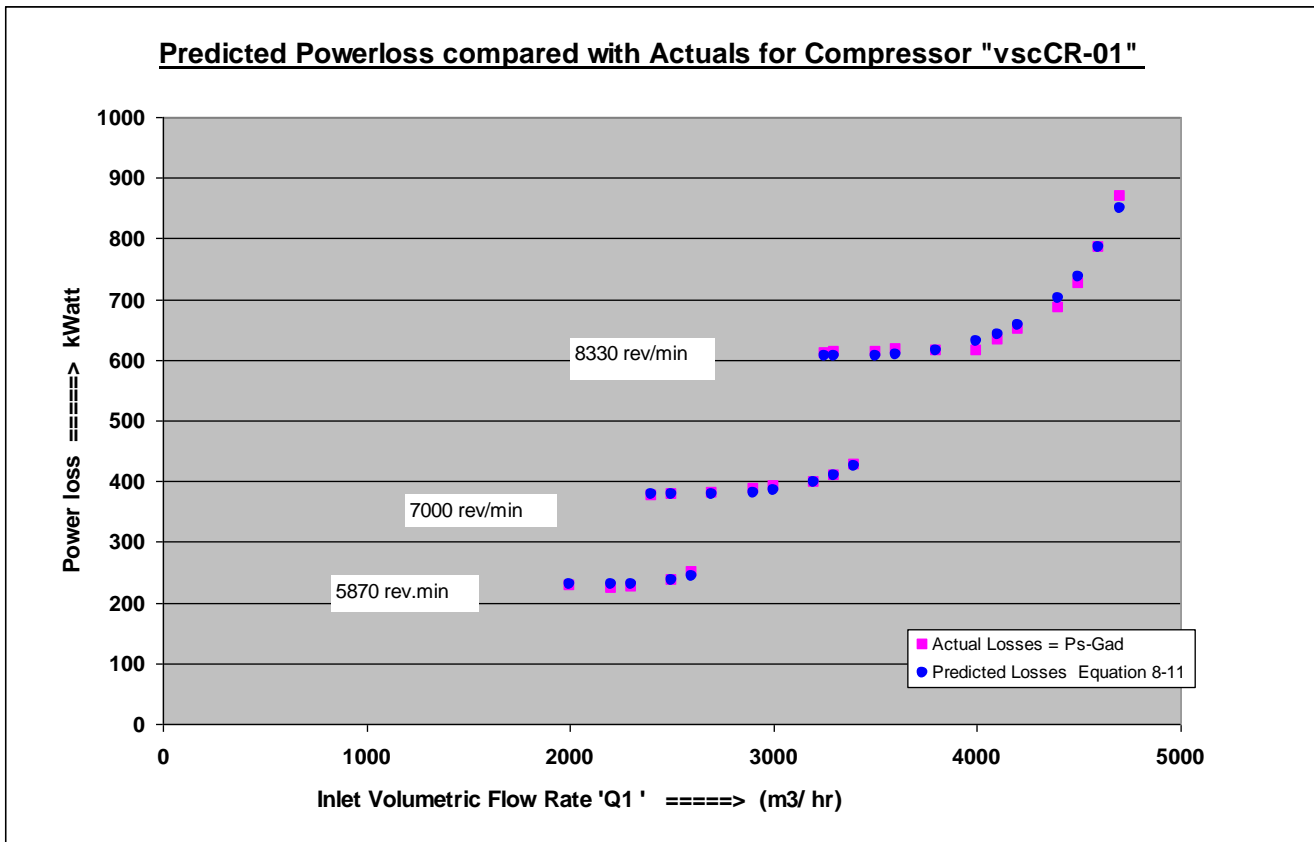
Diagram-8 Predicted Power Loss compared with Actuals for Compressor "vscCR-01".

To be precise: the overall average percentage error between predicted and actual loss data were calculated to be equal to 1.4 % !

Worth noting are the alternate ways in which eq.8-11 can be written:

$$Ps,loss / Fm = (K*fp) * (Q*)^2 * (S/1000)^2 \ldots\ldots(eq.8\text{-}11a)$$

In this equation the term 'Ps,loss/Fm' can be interpreted as the *enthalpy difference* 'DeltaH' associated with the energy being converted into heat as a result of loss processes during compressor operation!

$$DeltaH,loss / (S/1000)^2 = K * (Q*)^2 \ldots\ldots(eq.8\text{-}11b)$$

Note in eq.8-11b the analogy with the *reduced adiabatic head* 'H*',
being H* = Had /(S/1000)^2 , and note also that eq.8-11 can be written as:

$$Ps,loss = (K*fp) * (Q1)^3 * eps\text{-}h^2 * Rho1 * 1E\text{-}6 \ldots\ldots(eq.8\text{-}11c)$$

Here we see the dependence on the third power of the inlet volumetric flow rate!

9) <u>Predicting the Shaft Power 'Ps' in the simulation of "vscCR-01"</u>

To simulate the *total required shaft power* 'Ps' we now can use the predictive eq.8-11 to calculate the 'Ps,loss' values and add these to the adiabatic gas power 'Gad' to obtain the required shaft power 'Ps'. The resulting values when compared with the Manufacturer's quoted shaft power numbers show an average percentage error of 0.41 % ! Comparing this percentage error with the error obtained for the shaft power predictions by means of the adiabatic efficiency correlation of eq.7-1 a clear distinct improvement is achieved and not only that, we have found a description that provides some more insight into the *overall* loss process in addition! Thus:

Ps = Gad + Ps,loss   and with eq.8-11 we get

Ps = Had * Fm + (**K***fp*) * (Q*)^2 * Fm * (S/1000)^2

or alternatively written in reduced variables

Ps = (H* + H*loss) * Fm * (S/1000)^2

The simulated shaft power for compressor 'vscCR-01' can now be co-plotted together with the actual Manufacturer's quoted shaft power numbers as follows :
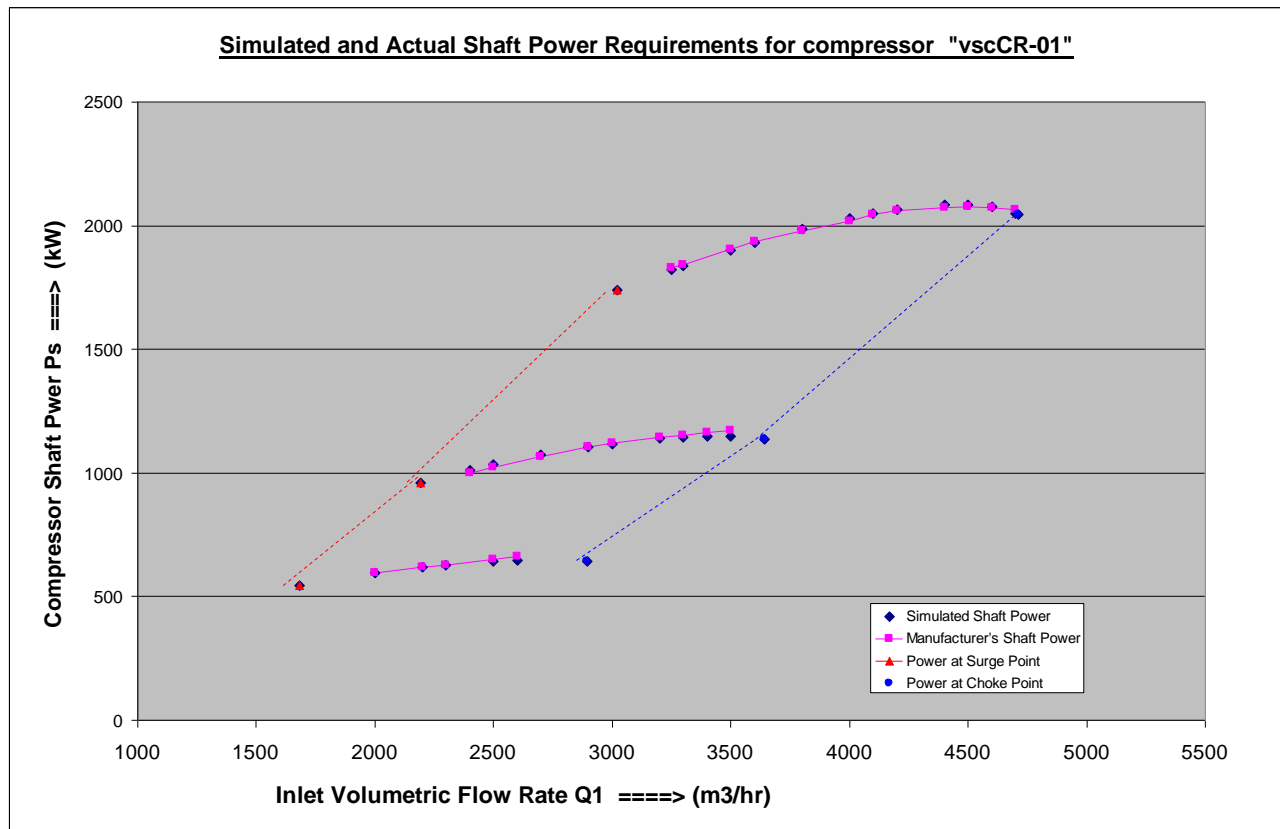
Diagram-9 Simulated and Actual Shaft Power for Compressor "vscCR-01" compared.

Note the surge- and choke points and lines drawn in this diagram as discussed in section 3 and 4.

Even though we do not need the Adiabatic Efficiency Factor anymore to be able to simulate the required shaft power 'Ps' it is interesting to see what the adiabatic efficiency factors are when we calculate these with the help of the newly developed loss (sub)-model equations as follows:

Eta-ad =   Gad / (Gad + Ps,loss)

The following Diagram shows the adiabatic efficiency factors calculated entirely from the Simulation Model is and in the following Diagram plotted side by side with efficiency factors calculated from the manufacturer's quoted total shaft power and the model's adiabatic Gas Power :

Simulated and Actual Adiabatic Efficiencies for compressor "vscCR-01"

Legend:
- Efficiency Factor from Simulated Ps,loss and Gad
- Efficiency Factor from Simulated Ps,loss and Gad
- Efficiency Factor from Simulated Ps,loss and Gad
- Efficiency Factor from Manufacturer's Ps and Gad

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Diagram-10 Simulated and Actual Adiabatic Efficiencies compared.

10) Upgrading the Compressor Simulation Excel Spreadsheet

The entire simulation can be carried out on a single worksheet of an Excel spreadsheet. An upgraded spreadsheet is presented here. The spreadsheet has been organized in three sections on the same worksheet as follows.

> Part 1: Input data : compressor Performance data ; Gas Quality data
> Part 2 (A): Analyzing the Manufacturers data: calculation of Q* and H*
> Part 2 (B) -1: 'Fingerprinting' the Performance Curves : H*fp
> Part 2 (B) -2: Determination of the Surge – and Choke Points and Lines
> Part 3: Simulation of Compressor vscCR-01 including inter- and
> extrapolated speeds (7800 and 8800 rpm);
> The calculation of Discharge Pressure; Pressure Ratio;
> Adiabatic Head; Gas Power; Power-Loss ;
> Required (total) Shaft Power and more.

==
PART 2. (A) "FINGER-PRINTING" THE COMPRESSOR'S CURVES: *CALCULATION OF "REDUCED" VARIABLES Q* (Q-star) and H* (H-star)*
<u>Analyzing the Manufacturer's data</u>   (surge and choke data copied from Part 2 (B))
==

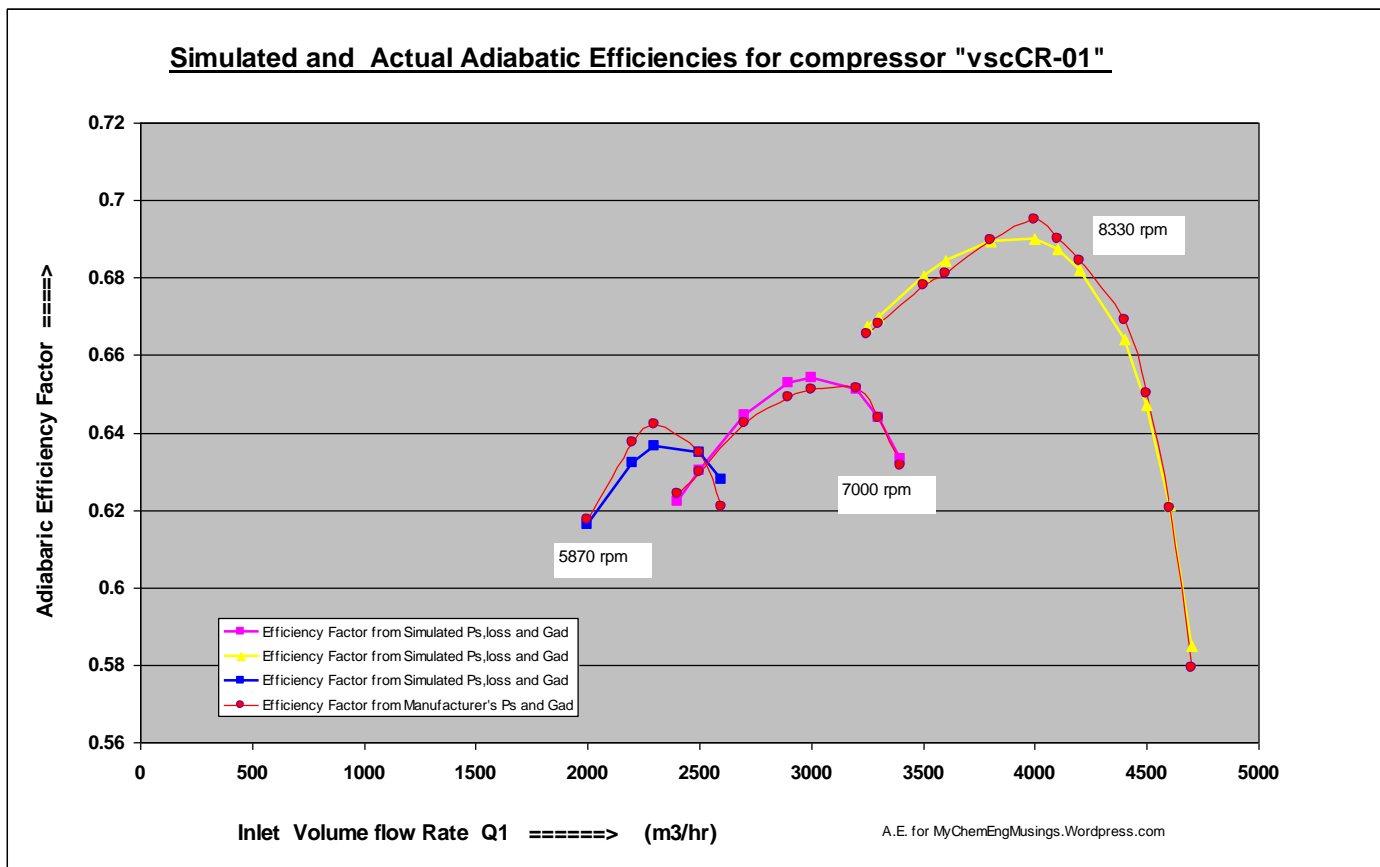|	S rev/min	Q1 m3/h	P2 barabs	P1 barabs	P2/P1 -----	Outlet Temp T2 oK	Outlet temp t2 oC	Outlet temp reduced T2red	Outlet Pressure reduced P2red	Outlet Compres. Factor Z2	Outlet Density Rho2 kg/m3	Z1 Manuf. -----	ICER Adiab Head Had kJ/kg	eps-h -----	Q*= Q1/S *eps-h	H*= Had/ (S/1000)^2
Calc Surge	7000	2192	24.657	10.5	2.34827	338.6	65.4	0.927	0.536	0.726	50.38	0.838	45.452	0.767	0.23998	0.92760
Curve#1	7000	2400	24.32	10.5	2.3162	337.9	64.8	0.925	0.529	0.728	49.61	0.838	44.677	0.771	0.264	0.9118
	7000	2500	24.17	10.5	2.3019	337.6	64.5	0.925	0.525	0.729	49.27	0.838	44.328	0.773	0.276	0.9047
	7000	2700	23.87	10.5	2.2733	337.1	63.9	0.923	0.519	0.732	48.59	0.838	43.625	0.776	0.299	0.8903
	7000	2900	23.39	10.5	2.2276	336.1	62.9	0.920	0.508	0.735	47.51	0.838	42.484	0.782	0.324	0.8670
	7000	3000	23.09	10.5	2.1990	335.5	62.3	0.919	0.502	0.738	46.84	0.838	41.760	0.786	0.337	0.8523
	7000	3200	22.36	10.5	2.1295	334.0	60.8	0.915	0.486	0.743	45.22	0.838	39.965	0.796	0.364	0.8156
	7000	3300	21.82	10.5	2.0781	332.8	59.7	0.911	0.474	0.747	44.03	0.838	38.605	0.803	0.379	0.7879
	7000	3400	21.23	10.5	2.0219	331.5	58.4	0.908	0.462	0.752	42.75	0.838	37.085	0.811	0.394	0.7568
	7000	3500	20.30	10.5	1.9333	329.4	56.3	0.902	0.441	0.759	40.75	0.838	34.614	0.824	0.412	0.7064
	0	0	0	0	0	0	0	0	0	0	0	0.838	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0.838	0	0		
Curve#1	0	0	0	0	0	0	0	0	0	0	0	0.838	0	0		
Eval. Choke	7000	3643.5	18.854	10.5	1.7956	326.0	52.9	0.893	0.410	0.770	37.70	0.838	30.573	0.845	0.44	0.62389
Calc Surge	8300	3020	34.193	10.5	3.25652	354.6	81.4	0.971	0.743	0.648	74.69	0.838	63.902	0.662	0.23998	0.92760
Curve#2	8330	3250	34.18	10.5	3.2552	354.6	81.4	0.971	0.743	0.648	74.65	0.838	64.344	0.662	0.258	0.9273
	8330	3300	34.03	10.5	3.2410	354.4	81.2	0.970	0.740	0.650	74.22	0.838	64.084	0.664	0.263	0.9235
	8330	3500	33.66	10.5	3.2057	353.8	80.7	0.969	0.732	0.653	73.17	0.838	63.438	0.667	0.280	0.9142
	8330	3600	33.39	10.5	3.1800	353.4	80.3	0.968	0.726	0.655	72.41	0.838	62.962	0.670	0.290	0.9074
	8330	3800	32.71	10.5	3.1152	352.4	79.2	0.965	0.711	0.661	70.53	0.838	61.750	0.677	0.309	0.8899
	8330	4000	31.86	10.5	3.0343	351.1	77.9	0.961	0.693	0.668	68.22	0.838	60.204	0.686	0.329	0.8676
	8330	4100	31.26	10.5	2.9771	350.1	77.0	0.959	0.680	0.673	66.62	0.838	59.092	0.692	0.341	0.8516
	8330	4200	30.5	10.5	2.9048	348.9	75.8	0.956	0.663	0.679	64.62	0.838	57.656	0.700	0.353	0.8309
	8330	4400	28.73	10.5	2.7362	346.0	72.8	0.948	0.625	0.693	60.12	0.838	54.189	0.719	0.380	0.7809
	8330	4500	27.46	10.5	2.6152	343.8	70.6	0.941	0.597	0.704	56.99	0.838	51.586	0.733	0.396	0.7434
	8330	4600	25.81	10.5	2.4581	340.8	67.6	0.933	0.561	0.717	53.06	0.838	48.045	0.753	0.416	0.6924
Curve#2	8330	4700	23.93	10.5	2.2790	337.2	64.0	0.923	0.520	0.731	48.72	0.838	43.766	0.776	0.438	0.6307
Eval. Choke	8300	4710.4	23.729	10.5	2.25993	336.8	63.6	0.922	0.516	0.733	48.27	0.838	43.292	0.778	0.44	0.62389
Calc Surge	5870	1681	19.3413	10.5	1.84203	327.2	54.0	0.896	0.420	0.766	38.72	0.838	31.962	0.838	0.23998	0.92760
Curve#3	5870	2000	19.2	10.5	1.82857	326.8	53.7	0.895	0.417	0.762	38.71	0.838	31.563	0.838	0.286	0.9160
	5870	2200	18.92	10.5	1.80190	326.2	53.0	0.893	0.411	0.770	37.83	0.838	30.762	0.844	0.316	0.8928
	5870	2300	18.7	10.5	1.78095	325.6	52.5	0.892	0.407	0.771	37.37	0.838	30.126	0.848	0.332	0.8743
	5870	2500	18.1	10.5	1.72381	324.1	51.0	0.888	0.393	0.776	36.13	0.838	28.358	0.857	0.365	0.8230
	5870	2600	17.7	10.5	1.68571	323.1	50.0	0.885	0.385	0.779	35.30	0.838	27.150	0.863	0.382	0.7880
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
Curve#3	0	0	0	0	0	0	0	0	0	0	0	0	0	0		
Eval. Choke	5870	2894.5	15.926	10.5	1.51680	318.3	45.2	0.872	0.346	0.793	31.68	0.838	21.498	0.892	0.44	0.62389

Diagram-11 Analyzing the Manufacturer's data: calculation of H* and Q*

===
Part 2 (B) COMPRESSOR'S FINGERPRINT PERFORMANCE CURVE : H*fp
plus reconstruction of SURGE points and line   plus CHOKE points and line
===

---- these three columns ---- for graphing			Q2= Fm/Rho2 m3/s
P2 surge	**P2** choke	**P2/P1** surge	
24.657		2.348274	0.25
			0.28
			0.30
			0.32
			0.36
			0.37
			0.41
			0.44
			0.46
			0.500
	18.854		**0.563**
34.193		3.256522	0.24
			0.25
			0.26
			0.28
			0.29
			0.31
			0.34
			0.36
			0.38
			0.43
			0.46
			0.50
			0.56
	23.729		**0.568**
19.341		1.842031	0.253
			0.301
			0.339
			0.358
			0.403
			0.429
	15.926		**0.532**

(B1)   Determining the Fingerprint performance curve H*fp as funtion of Q*
All values of **H*** and **Q*** calculated from the manuf data are plotted in one graph
The **H* vs Q*** curves for the three speeds should have closely "coalesced" into one single curve
The data of curve #2 was selected for a curve fit using quadratic regression as curve#2
which has most data points describing the curvature in detail
An "in-chart" quadratic regression was done and the coefficients copied (manually) to
the speadsheet See CHART = A =

$$H*fp = A * (Q*)^2 + B * (Q*) + C$$

Coefficients : A = -7.5914   B = 3.6436   C= 0.4904

(B2)   Determining the Surge Point and Surge line
The Surge Point on the H*fp curve can be 'reconstructed' assuming that surge occurs
when the H*fp curve reaches its maximum value; in other words the slope of the curve =0
maximum found by differentiation of quadratic curve yields:

Q*surge= - B / 2A = 0.23998   H*surge= - (B)^2 / 4A + C = 0.92760

Reconstructing the **Surge points** (and line) for the three different speeds:
  Calculating **P2/P1 at surge** for speeds: Calc of **Q1 at surge** for speeds:
S (rev/m)= 8330   **3.25652**   S= 8330 rev/min :   **3019.8** m3/h
S (rev/m)= 7000   **2.34827**   S= 7000   **2191.6** m3/h
S (rev/m)= 5870   **1.84203**   S= 5870   **1680.9** m3/h

3020.294
2191.34
(B3)   Evaluating the Choke point and Choke line   1684.195

Choke occurs at high flow rates where internal , local velocity in rotor or stator
approaches or reaches the velocity of sound pertinent to the gas quality under the
high flow high pressure operating conditions.   For example the velocity
of sound in propane at 20 Bar abs and 85 Degr.C equals 225 m/s (Ref …….)
A high end limitation of internal velocities would translate into a limit on
the Volumetric flow rate (Q2) under discharge (P2 , T2) conditions at a given speed.See Note
As performance curve at high Q* runs into the "stonewall" Choke, the curve suddenly drops steeply
It is interesting to look how the slope of the H*fp curve varies in terms of the angle
and how it approaches - 90 degrees

slope of H*fp curve = 2A * (Q*) + B

Points along the H*fp curve

point	Q*	slope	angle (degrees)
Surge	0.23998	0	0
point 1	0.258	-0.28	-16
point 5	0.309	-1.04	-46
mid-range	0.3480	-1.64	-59
point 9	0.380	-2.12	-65
point 11	0.416	-2.67	-69
point 12	0.438	-3.00	-72
Assumed Choke	**0.440**	-3.04	**-72**
H*choke =	**0.62389**		

Reconstructing the **Choke points** (and line) for the three different speeds:
  Calculating **P2/P1 at choke** for speeds: Calc of **Q1 at choke** for speeds:
S (rev/m)= 8330   **2.25993**   S= 8300 rev/min :   **4710.4** m3/h
S (rev/m)= 7000   **1.79560**   S= 7000   **3643.5** m3/h
S (rev/m)= 5870   **1.51680**   S= 5870   **2894.5** m3/h

Note   Observe the values of the volume flow *Q2 at high Q1 vol.flowrates: they are* approaching a
of around **0.5 m3/s**   (higher speeds very close together) .
Meaning that irrespective of speed or discharge pressure level the max volume flowrate Q2
abt 0.55 m3/s Likely indicatng that higher local velocities inside machine approaching a limi
See Q2 volume flows calculated and shown in column "U"
Have assumed here that highest operable point of volumeflow on the curve equal to or close

Diagram-12 Fingerprinting the H – Q curves : determine H*fp ; plus reconstructing
the surge points and choke points and lines.

==

PART 3   SIMULATION OF THE EXISTING COMPRESSOR' S PERFORMANCE : CALCULATING THE PERFORMANCE AT DIFFERENT SPEEDS :
Calculating : Outlet Pressure ; Pressure Ratio ; Adiabatic Head ; GasPower ; Power Loss ; Total Required Shaft Power

==

Characteristic data for the Fingerprint of this compressor machine Refer to Part 2 ABC ; plus summary Inlet Conditions & Gas Quality

=1=	**H*fp =**	**A * (Q*)^2 + B * (Q*) + C**		**A =**	-7.5914		**B =**	3.6436	**C =**	0.4904					
=2=	**Surge Point:**	**H*fp-surge =**	0.92760			**Q*-surge=**	0.23998								
=3=	**Choke Point: H*fp-choke =**	0.62389				**Q*-choke=**	0.44								
=4=	**Adiabatic efficiency characteristic function of Q***					Etad * (S/1000)^0.39 /(Q1/S) = A * (Q*) + B						A=	-8.4625	B=	6.083
=5=	**Inlet Conditions**	**P1 (Barabs) =**		10.5	**t 1 (oC) =**	27	**Gas Density (kg/m3)**		20.96	Ad Compr exponent		0.141189			
=6=	**Gas Quality**		**MW =**	41.74	**Z1=**	0.838	**Cp/Cv=**	1.1644	**Pcrit=**	46	**Tcrit=**	365.15	**Zc =**		

0.285

S rev/min	Q1 m3/h	Fm kg/s	eps-h -------	Q* =Q1/S * eps-h	H*fp -------	Had kJ/kg	P2/P1 calc	P2 calc	T2 oK	Pred -------	T2red -------	Z 2 -------	P2 Manufact Barabs	P2 calc Error% avg=	Eta-ad calc -------	Pshaft calc kW	Pshaft Manuf kW	Error% avg=
7000	2191.6	12.760	0.7665	0.23998	0.92760	45.452	2.34827	24.657	338.60	0.5360	0.92728	0.726		0.51				0.98
7000	3643.5	21.213	0.8453	0.44	0.62389	30.571	1.79560	18.854	326.01	0.4099	0.89281	0.770						
7000	2400	13.973	0.768	0.2632	0.9235	45.252	2.340	24.57	338.42	0.5341	0.92681	0.726	24.32	1.02	0.619	1022	1000	2.16
7000	2500	14.556	0.769	0.2746	0.9185	45.006	2.330	24.46	338.22	0.5318	0.92624	0.727	24.17	1.20	0.629	1042	1024	1.78
7000	2700	15.720	0.773	0.2983	0.9018	44.188	2.296	24.11	337.52	0.5241	0.92435	0.730	23.87	1.00	0.643	1081	1067	1.30
7000	2900	16.884	0.780	0.3233	0.8749	42.870	2.243	23.55	336.41	0.5120	0.92129	0.734	23.39	0.69	0.649	1115	1105	0.90
7000	3000	17.467	0.785	0.3365	0.8569	41.990	2.208	23.18	335.67	0.5040	0.91925	0.737	23.09	0.41	0.649	1130	1120	0.86
7000	3200	18.631	0.797	0.3645	0.8099	39.684	2.119	22.25	333.72	0.4836	0.91391	0.744	22.36	0.51	0.642	1152	1143	0.80
7000	3300	19.213	0.805	0.3796	0.7796	38.200	2.063	21.66	332.46	0.4709	0.91048	0.749	21.82	0.73	0.634	1158	1152	0.56
7000	3400	19.796	0.815	0.3956	0.7437	36.440	1.998	20.98	330.97	0.4561	0.90640	0.754	21.23	1.16	0.622	1160	1162	0.18
7000	3500	20.378	0.826	0.4128	0.7009	34.343	1.924	20.20	329.20	0.4391	0.90154	0.760	20.30	0.50	0.606	1154	1172	1.50
8330	3019.8	17.582	0.6620	0.23998	0.92760	64.365	3.25652	34.193	354.60	0.7433	0.97110	0.648						
8330	4710.4	27.425	0.7781	0.44	0.62389	43.291	2.25993	23.729	336.77	0.5159	0.92228	0.733						
8330	3250	18.922	0.663	0.2587	0.9249	64.181	3.246	34.08	354.43	0.7409	0.97065	0.649	34.18	0.28	0.665	1827	1829	0.09
8330	3300	19.213	0.664	0.2629	0.9236	64.089	3.241	34.03	354.36	0.7398	0.97044	0.650	34.03	0.00	0.669	1841	1843	0.08
8330	3500	20.378	0.667	0.2802	0.9153	63.513	3.210	33.70	353.87	0.7326	0.96911	0.652	33.66	0.12	0.682	1897	1906	0.47
8330	3600	20.960	0.669	0.2893	0.9092	63.086	3.186	33.46	353.51	0.7273	0.96812	0.654	33.39	0.20	0.687	1924	1937	0.67
8330	3800	22.124	0.676	0.3084	0.8921	61.900	3.123	32.79	352.51	0.7129	0.96537	0.660	32.71	0.25	0.693	1976	1980	0.21
8330	4000	23.289	0.686	0.3293	0.8670	60.159	3.032	31.83	351.03	0.6920	0.96134	0.668	31.86	0.08	0.692	2023	2017	0.32
8330	4100	23.871	0.692	0.3407	0.8506	59.020	2.973	31.22	350.07	0.6787	0.95870	0.673	31.26	0.13	0.689	2045	2044	0.04
8330	4200	24.453	0.700	0.3529	0.8309	57.652	2.904	30.50	348.91	0.6630	0.95553	0.679	30.5	0.01	0.683	2064	2060	0.19
8330	4400	25.618	0.720	0.3804	0.7779	53.977	2.726	28.62	345.80	0.6222	0.94702	0.694	28.73	0.37	0.662	2090	2074	0.75
8330	4500	26.200	0.734	0.3965	0.7416	51.459	2.609	27.40	343.67	0.5956	0.94119	0.704	27.46	0.23	0.645	2092	2078	0.65
8330	4600	26.782	0.752	0.4150	0.6950	48.223	2.466	25.89	340.94	0.5628	0.93369	0.716	25.81	0.31	0.621	2080	2073	0.32
8330	4700	27.364	0.775	0.4374	0.6318	43.839	2.282	23.96	337.23	0.5209	0.92354	0.731	23.93	0.12	0.588	2041	2067	1.27
5870	1680.9	9.786	0.8381	0.23998	0.92760	31.962	1.84203	19.341	327.19	0.4205	0.89603	0.766						
5870	2894.5	16.852	0.8923	0.44	0.62389	21.497	1.51680	15.926	318.33	0.3462	0.87179	0.793						
5870	2000	11.644	0.841	0.2866	0.9111	31.395	1.823	19.14	326.70	0.4161	0.89471	0.768	19.2	0.31	0.625	585	595	1.69
5870	2200	12.809	0.846	0.3172	0.8824	30.404	1.790	18.80	325.87	0.4086	0.89242	0.771	18.92	0.66	0.639	610	618	1.35
5870	2300	13.391	0.850	0.3330	0.8619	29.698	1.767	18.55	325.27	0.4033	0.89078	0.772	18.7	0.79	0.641	620	628	1.28
5870	2500	14.556	0.860	0.3662	0.8067	27.796	1.706	17.91	323.66	0.3894	0.88638	0.777	18.1	1.04	0.637	635	650	2.33
5870	2900	15.138	0.866	0.3837	0.7709	26.561	1.667	17.51	322.62	0.3806	0.88352	0.781	17.7	1.09	0.630	638	662	3.58
7800	2649.1	15.424	0.7066	0.23998	0.92760	56.435	2.84452	29.867	347.89	0.6493	0.95273	0.684						
7800	4255.4	24.776	0.8065	0.44	0.62389	37.957	2.05404	21.567	332.26	0.4689	0.90992	0.749						
7800	2850	16.593	0.707	0.2585	0.9250	56.277	2.837	29.78	347.75	0.6475	0.95235	0.685			0.639	1462		
7800	3100	18.049	0.711	0.2827	0.9138	55.593	2.803	29.43	347.17	0.6399	0.95076	0.688			0.658	1524		
7800	3100	18.049	0.711	0.2827	0.9138	55.593	2.803	29.43	347.17	0.6399	0.95076	0.688			0.658	1524		
7800	3400	19.796	0.721	0.3141	0.8859	53.896	2.722	28.58	345.74	0.6214	0.94683	0.695			0.670	1592		
7800	3700	21.542	0.737	0.3497	0.8363	50.878	2.583	27.12	343.18	0.5896	0.93984	0.706			0.665	1648		
7800	4000	23.289	0.765	0.3924	0.7513	45.707	2.359	24.77	338.81	0.5384	0.92786	0.725			0.636	1674		
7800	4100	23.871	0.779	0.4093	0.7100	43.198	2.256	23.69	336.69	0.5150	0.92205	0.733			0.618	1669		
7800	4200	24.453	0.795	0.4283	0.6585	40.061	2.133	22.40	334.03	0.4869	0.91479	0.743			0.594	1649		
8800	3414.9	19.882	0.6184	0.23998	0.92760	71.833	3.69038	38.749	360.91	0.8424	0.98840	0.608						
8800	5155.4	30.016	0.7511	0.44	0.62389	48.314	2.46979	25.933	341.02	0.5638	0.93391	0.716						
8800	3700	21.542	0.620	0.2607	0.9244	71.582	3.675	38.58	360.69	0.8388	0.98780	0.610			0.698	2209		
8800	3900	22.707	0.623	0.2761	0.9177	71.066	3.643	38.25	360.26	0.8316	0.98660	0.613			0.711	2270		
8800	4200	24.453	0.632	0.3014	0.8989	69.612	3.556	37.34	359.03	0.8117	0.98324	0.621			0.722	2358		
8800	4500	26.200	0.647	0.3308	0.8649	66.981	3.403	35.73	356.80	0.7767	0.97714	0.635			0.719	2441		
8800	4700	27.364	0.663	0.3542	0.8286	64.166	3.245	34.08	354.42	0.7408	0.97062	0.649			0.706	2488		
8800	4900	28.529	0.688	0.3830	0.7724	59.814	3.014	31.65	350.74	0.6880	0.96054	0.670			0.678	2518		
8800	5100	29.693	0.731	0.4237	0.6714	51.996	2.634	27.66	344.13	0.6012	0.94243	0.702			0.620	2491		

Diagram-13 Copy of Worksheet image with the Simulation of Compressor "vscCR-01" performance curves for five speeds including interpolated and extrapolated speeds.

Note that this part of the spreadsheet shows the shaft power calculation with the original correlation of Eta-ad (shown at the top of worksheet image). The power loss calculations as explained in this document in section 8 are performed on a separate part of the worksheet not shown here.

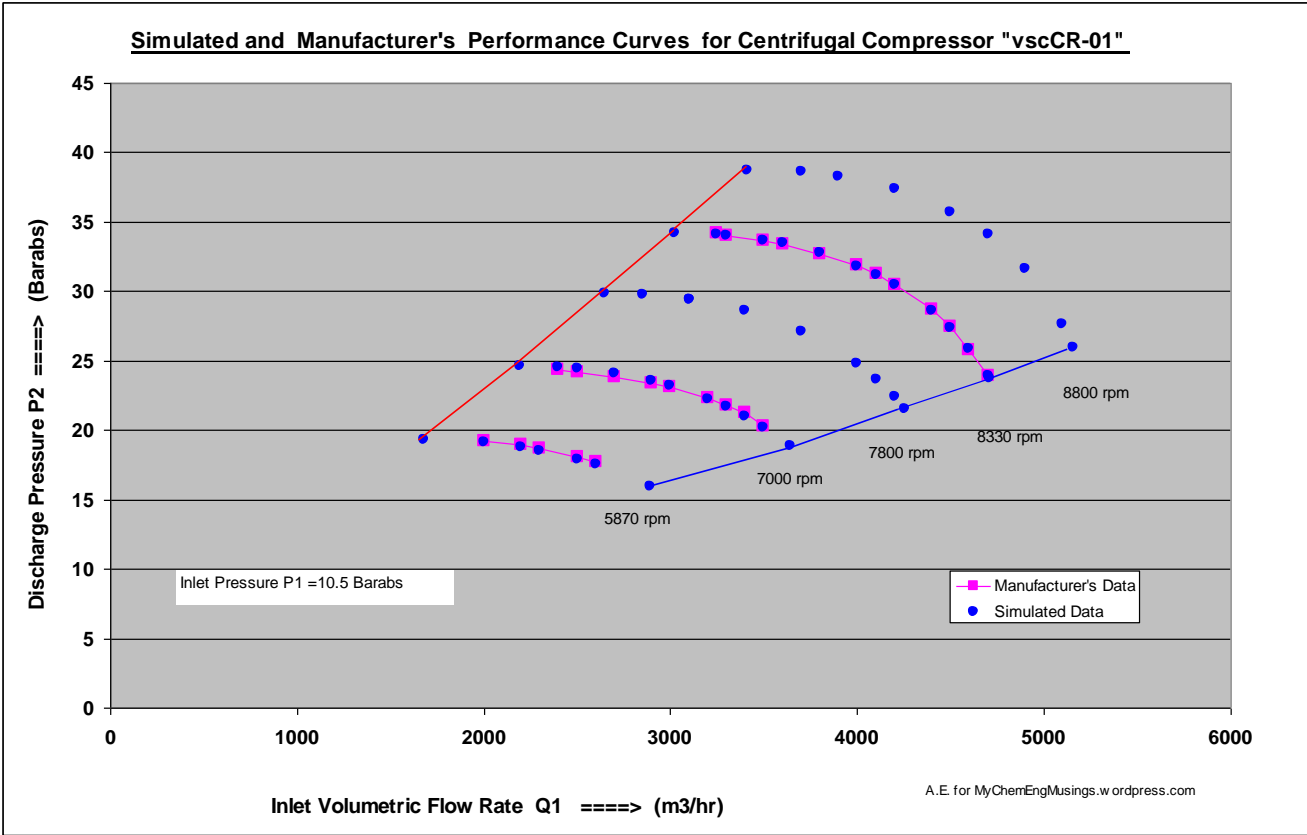The Simulated discharge pressures shown in Diagram 13 have been plotted in the next Diagram-14.

Diagram-14 Simulated and Manufacturer's Discharge Pressures for Centrifugal Compressor "vscCR-01" for 5 speeds. Note the Surge line as defined in Section 3 and the Choke Line as in Section 4.
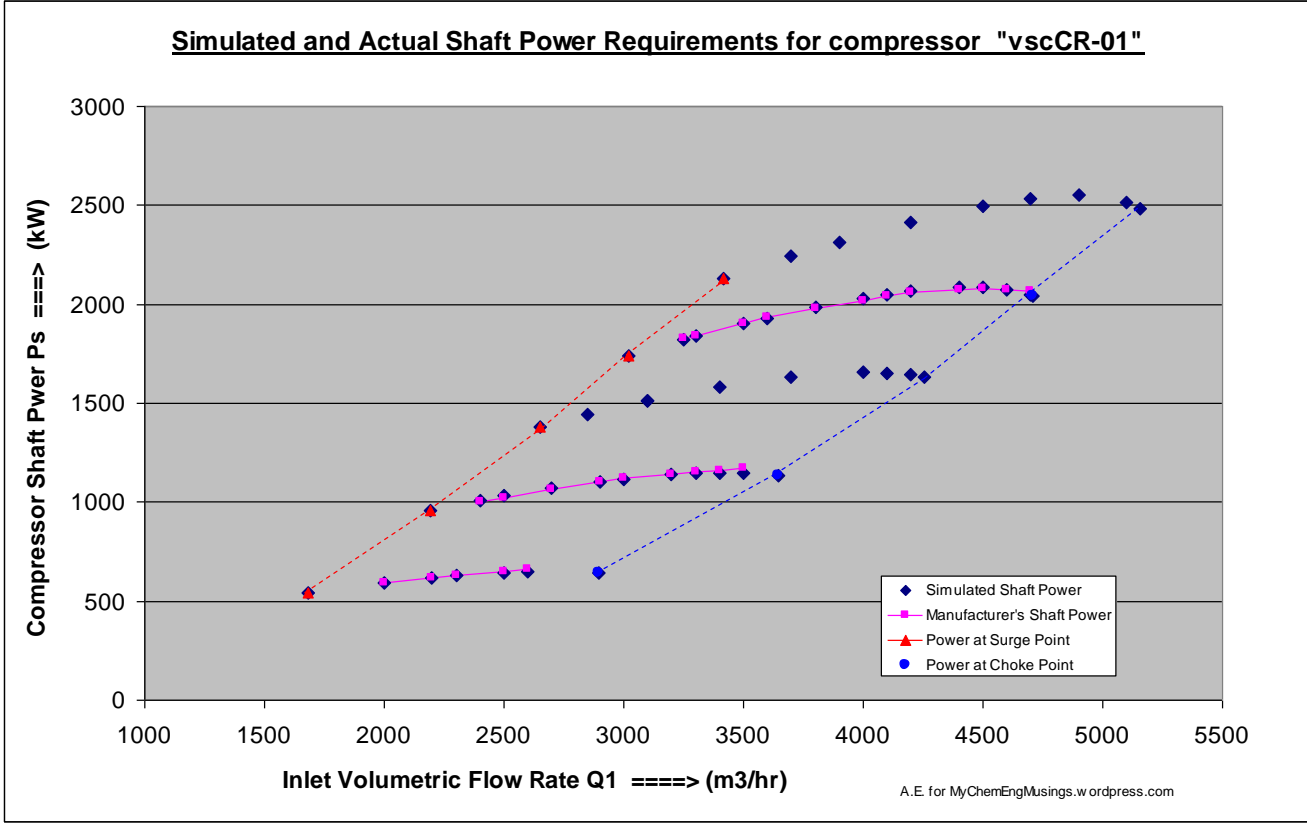
Diagram-15 Simulated and Manufacturer's Shaft Power of Compressor "vscCR-01" for 5 speeds.

The spreadsheet Simulation Model of variable speed compressor "vscCR-01" and its calculation intermediary and final calculations results shown in Diagram-13 allows many other graphs and different types of Performance Maps to be plotted easily, Diagram 14 and 15 are two of the examples.

End Post -------------- June 2020